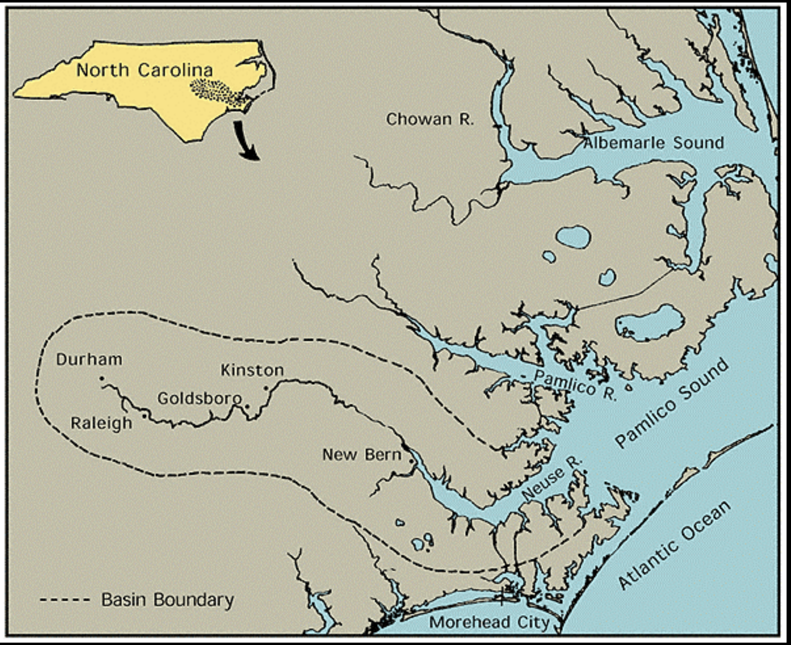
Indirect Effects and Distributed Control in Ecosystems

5. Distributed Control in the Environ Networks of a Seven-Compartment Model of Nitrogen Flow in the Neuse River Estuary, USA: Steady-State Analysis



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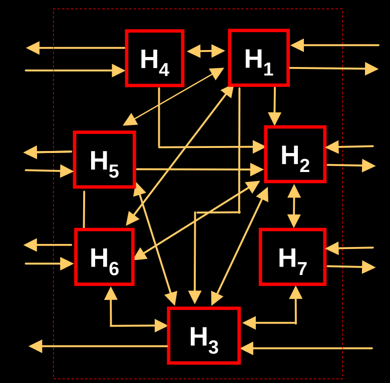
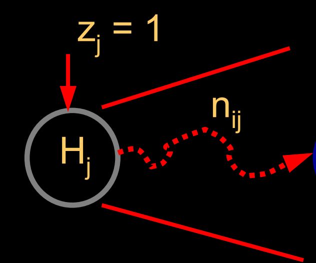
Objectives



Introduction

- Review the “Control Matrix” of a cold water spring energy model
- Applicability to the Neuse River Estuary nitrogen model



Methods

- Consider pair-wise relations between two components in a network
- Control theory and equation development

$$[T]^{out}_{n\times1} = [N]_{n\times n} \times [z]_{n\times1}$$

Results

- *Control Ratio*, cr_{ij}
- *Control Difference*, cd_{ij}
- *System Control*, cs_i

$$CR\times10 = \begin{bmatrix} 0 & -0.1 & -3.3 & 0.1 & 2.8 & 0.2 & -0.8 \\ 0.1 & 0 & -2.9 & 0.4 & 2.4 & -.2 & -0.7 \\ 3.3 & 2.9 & 0 & 3.3 & 4.4 & 2.9 & 3.4 \\ -0.1 & -0.4 & -3.3 & 0 & 2.4 & -0.3 & -1.1 \\ -2.8 & -2.4 & -4.4 & -2.4 & 0 & -2.1 & -2.8 \\ -0.2 & 0.2 & -2.9 & 0.3 & 2.1 & 0 & -0.5 \\ 0.8 & 0.7 & -3.4 & 1.1 & 2.8 & 0.5 & 0 \end{bmatrix}$$

$$CD\times10 = \begin{bmatrix} 0 & -0.2 & -4.1 & 0.2 & 3.6 & 0.3 & -1.0 \\ 0.2 & 0 & -3.6 & 0.4 & 3.0 & -0.3 & -0.9 \\ 4.1 & 3.6 & 0 & 3.9 & 5.5 & 3.6 & 4.4 \\ -0.2 & -0.4 & -3.9 & 0 & 2.9 & -0.3 & -1.3 \\ -3.6 & -3.0 & -5.5 & -2.9 & 0 & -2.6 & -3.5 \\ -0.3 & 0.3 & -3.6 & 0.3 & 2.6 & 0 & -0.7 \\ 1.0 & 0.9 & -4.4 & 1.3 & 3.5 & 0.7 & 0 \end{bmatrix}$$

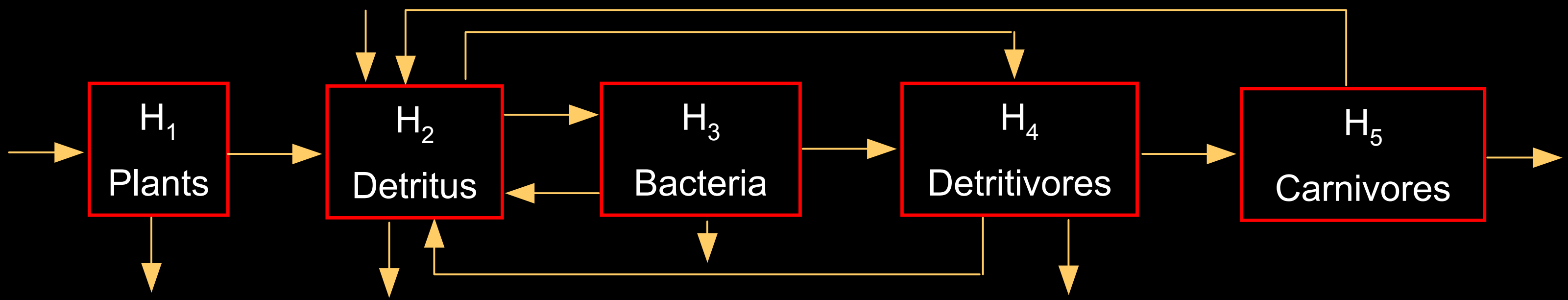
$$cs_i \times 10 = \begin{bmatrix} -1.3 \\ -1.2 \\ 25.2 \\ -3.2 \\ -21.2 \\ -1.3 \\ 3.0 \end{bmatrix}$$

Conclusions

Introduction (1)

Steady-state energy flow model of a cold water spring ecosystem

(Patten et al. 1976)



	H_1	H_2	H_3	H_4	H_5
H_1	1.00/1.00 1 → **0**	0.00/0.93 0 → **1**	0.00/0.93 0 → **1**	0.00/0.93 0 → **1**	0.00/0.93 0 → **1**
H_2	0.96/0.00 ∞ → **0**	1.21/1.21 1 → **0**	0.37/1.21 0.31 → **0.69**	0.19/1.21 0.15 → **0.85**	0.54/1.21 0.45 → **0.55**
H_3	0.43/0.00 ∞ → **0**	0.56/0.17 3.34 → **0**	1.17/1.17 1 → **0**	0.08/0.20 0.42 → **0.58**	0.25/0.20 1.29 → **0**
H_4	0.20/0.00 ∞ → **0**	0.25/0.04 6.44 → **0**	0.09/0.40 2.36 → **0**	1.04/1.04 1 → **0**	0.11/1.04 0.11 → **0.89**
H_5	0.03/0.00 ∞ → **0**	0.03/0.02 2.17 → **0**	0.01/0.20 0.78 → **0.22**	0.16/0.02 8.94 → **0**	1.02/1.02 1 → **0**

(Patten and Auble 1981)

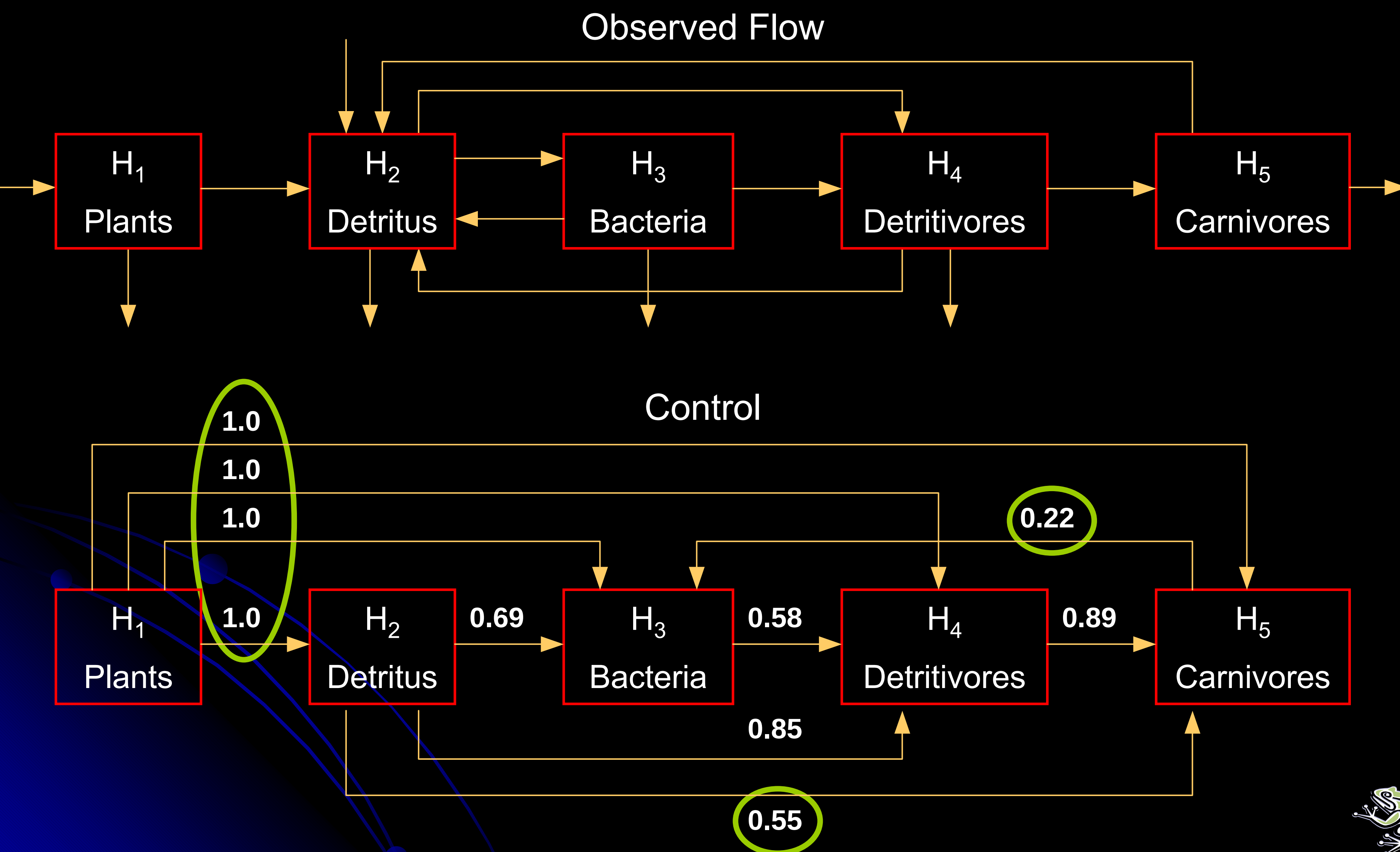
Control Matrix,

format for entries is:

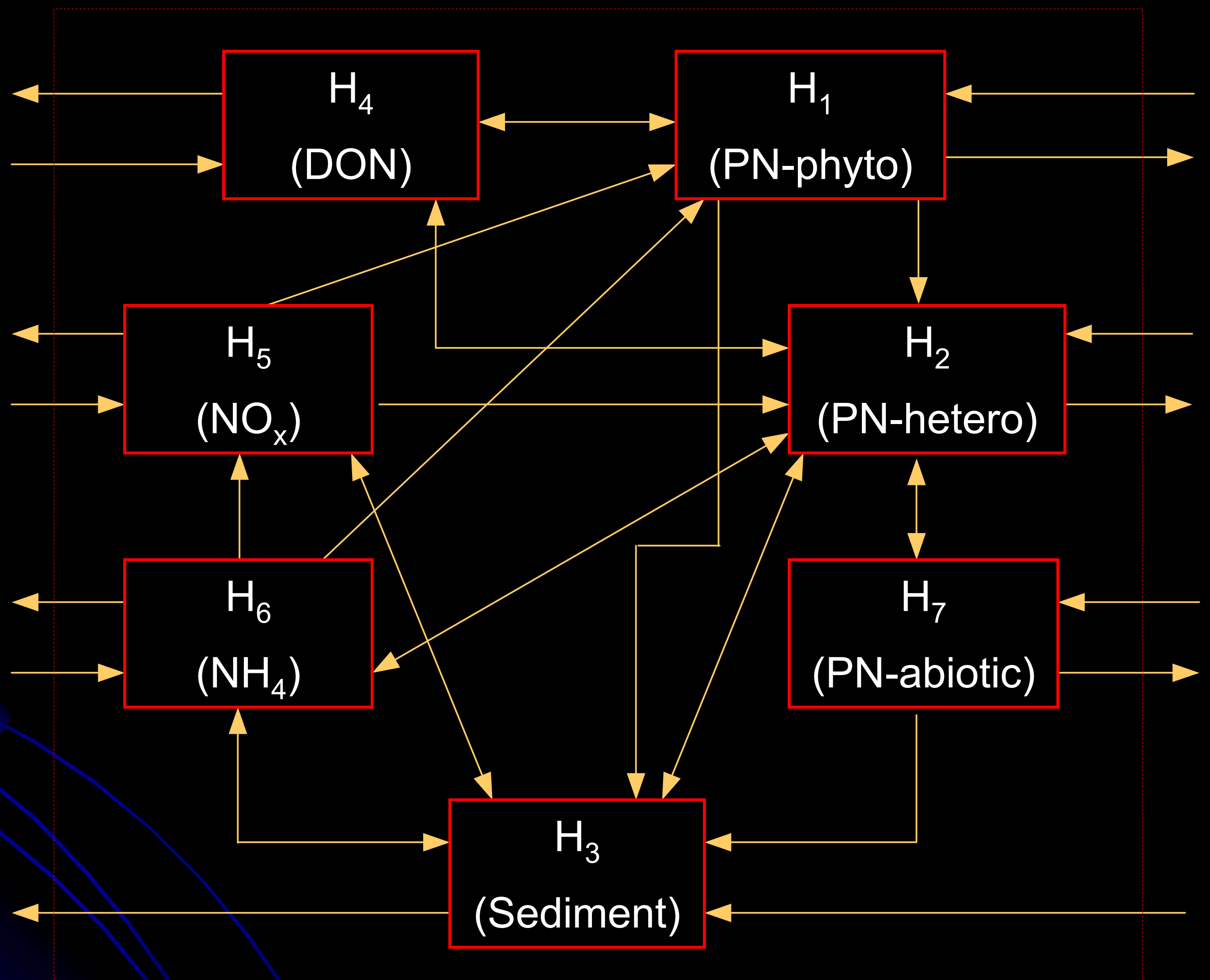
n_{ij}/n'_{ji}

where N and N' are the transitive closure matrices

Introduction (2)



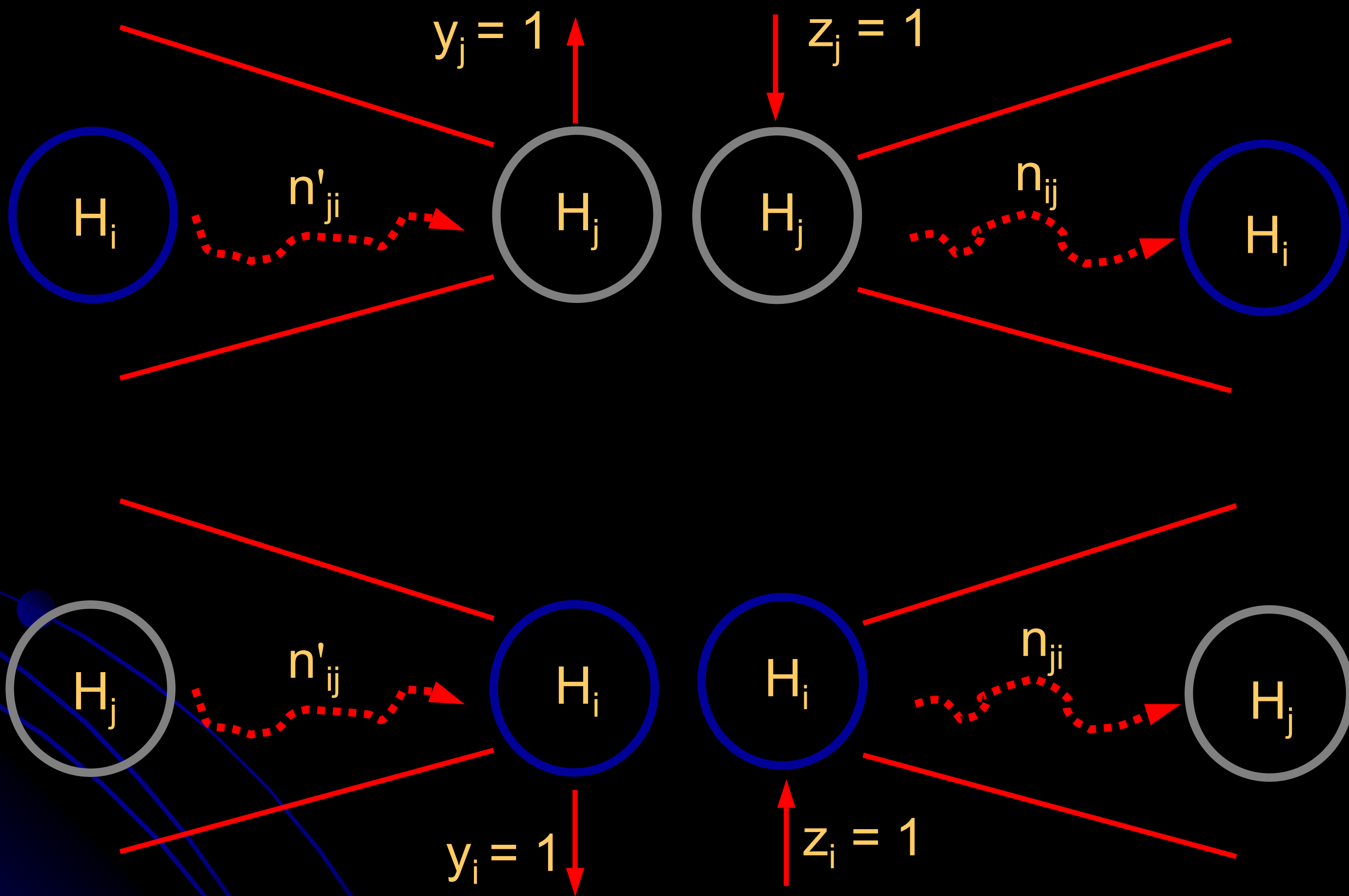
Introduction (3)



Nitrogen flow digraph in the Neuse River Estuary
(Christian and Thomas 2003)

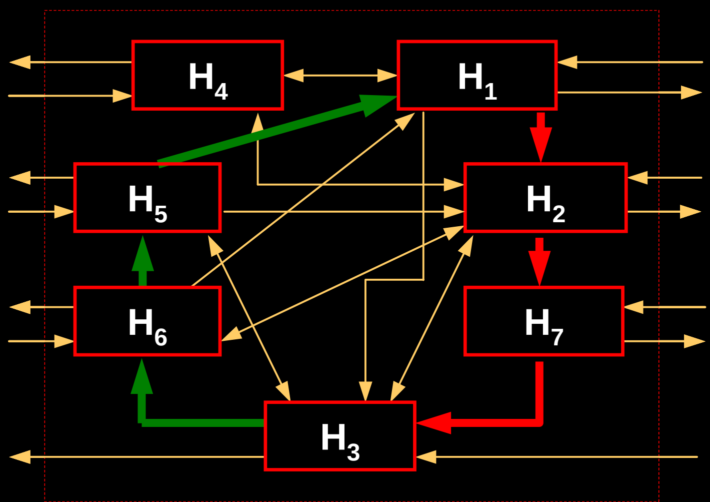
Methods (1)

Pair-wise environ relations



Methods (2)

Powers of the Adjacency Matrix, A^m, quantifies number of pathways between components



Example: Quantity of paths of length, m = 3, between H_1 and H_3

$$A^3 = \begin{bmatrix} 4 & 3 & 3 & 3 & 5 & 5 & 4 \\ 8 & 7 & 9 & 7 & 9 & 11 & 6 \\ 8 & 9 & 7 & 4 & 7 & 9 & 5 \\ 4 & 6 & 4 & 1 & 2 & 4 & 1 \\ 4 & 3 & 4 & 3 & 4 & 5 & 3 \\ 5 & 6 & 5 & 3 & 4 & 6 & 2 \\ 4 & 6 & 4 & 1 & 2 & 4 & 1 \end{bmatrix}$$

$$\left[A^T\right]^3 = \begin{bmatrix} 4 & 8 & 8 & 4 & 4 & 5 & 4 \\ 3 & 7 & 9 & 6 & 3 & 6 & 6 \\ 3 & 9 & 7 & 4 & 4 & 5 & 4 \\ 3 & 7 & 4 & 1 & 3 & 3 & 1 \\ 5 & 9 & 7 & 2 & 4 & 4 & 2 \\ 5 & 11 & 9 & 4 & 5 & 6 & 4 \\ 4 & 6 & 5 & 1 & 3 & 2 & 1 \end{bmatrix}$$

8 Pathways from H_1 forward to H_3

3 Pathways from H_1 back to H_3

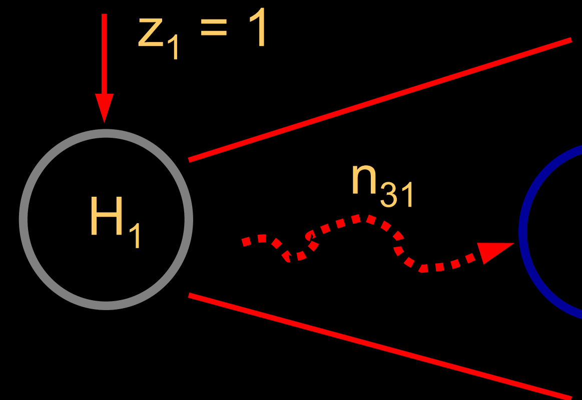
$$a_{ij}^{(m)} \neq a_{ij}^{T(m)} \qquad m = \text{pathway length} = 1, 2, \ldots, \infty$$

Methods (3)

Open-loop Control

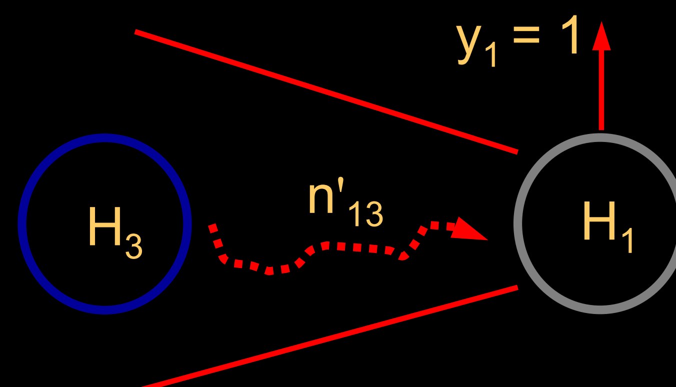
$$T_3^{out} = [n_{31} \times z_1]^{in} + [n_{32} \times z_2]^{in} + ... + [n_{3n} \times z_n]^{in}$$

$$T_{31}^{in} = n_{31} \times z_1$$



$$T_3^{in} = [y_1 \times n'_{31}]^{out} + [y_2 \times n'_{32}]^{out} + ... + [y_n \times n'_{3n}]^{out}$$

$$T_{13}^{out} = y_1 \times n'_{13}$$



Methods (4)

Fractional Transfer Coefficient, η

$$\eta_{ij} = \left[\frac{n_{ij}}{T_i^{out}} \times z_j\right] = \left[y_i \times \frac{n'_{ij}}{T_j^{in}}\right]$$

when $z_j = y_i = 1$

$$\eta_{ji} = \left[y_j \times \frac{n'_{ji}}{T_i^{in}}\right] = \left[\frac{n_{ji}}{T_j^{out}} \times z_i\right]$$

when $z_i = y_j = 1$

Control Ratio

$$CR = cr_{ij} = \frac{\eta_{ij} - \eta_{ji}}{\max(\eta_{ij}, \eta_{ji})}$$

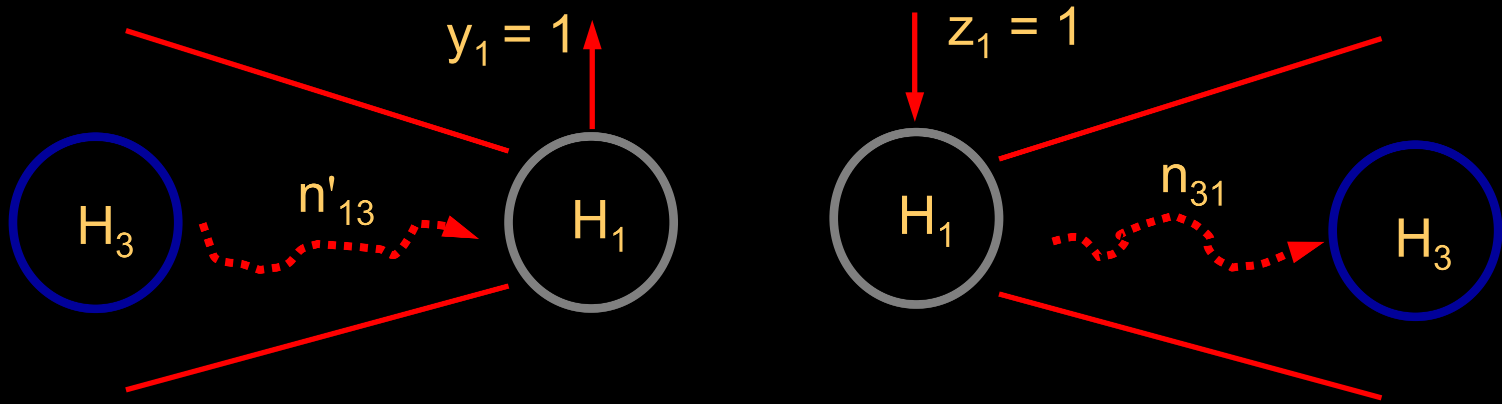
Control Difference

$$CD = cd_{ij} = \eta_{ij} - \eta_{ji}$$

System Control vector

$$cs_i = \sum_{k=1}^{n} cd_{ik} \qquad n = \text{number of system components}$$

Results (1)



Control Ratio

$$\eta \times 1000 = \begin{bmatrix} 1.36 & 1.26 & 0.85 & 1.24 & 1.29 & 1.30 & 1.15 \\ 1.28 & 1.34 & 0.88 & 1.25 & 1.28 & 1.30 & 1.22 \\ 1.26 & 1.24 & 1.63 & 1.19 & 1.26 & 1.25 & 1.31 \\ 1.23 & 1.21 & 0.80 & 1.53 & 1.19 & 1.21 & 1.10 \\ 0.93 & 0.97 & 0.71 & 0.91 & 1.56 & 1.00 & 0.90 \\ 1.27 & 1.33 & 0.89 & 1.24 & 1.27 & 1.38 & 1.21 \\ 1.26 & 1.32 & 0.87 & 1.23 & 1.25 & 1.28 & 1.67 \end{bmatrix}$$

$$CR = \begin{bmatrix} 0 & -0.01 & -0.33 & 0.01 & 0.28 & 0.02 & -0.08 \\ 0.01 & 0 & -0.29 & 0.04 & 0.24 & -0.02 & -0.07 \\ 0.33 & 0.29 & 0 & 0.33 & 0.44 & 0.29 & 0.34 \\ -0.01 & -0.04 & -0.33 & 0 & 0.24 & -0.03 & -.11 \\ -0.28 & -0.24 & -0.44 & -0.24 & 0 & -0.21 & -0.28 \\ -0.02 & 0.02 & -0.29 & 0.03 & 0.21 & 0 & -0.05 \\ 0.08 & 0.07 & -0.34 & 0.11 & 0.28 & 0.05 & 0 \end{bmatrix}$$

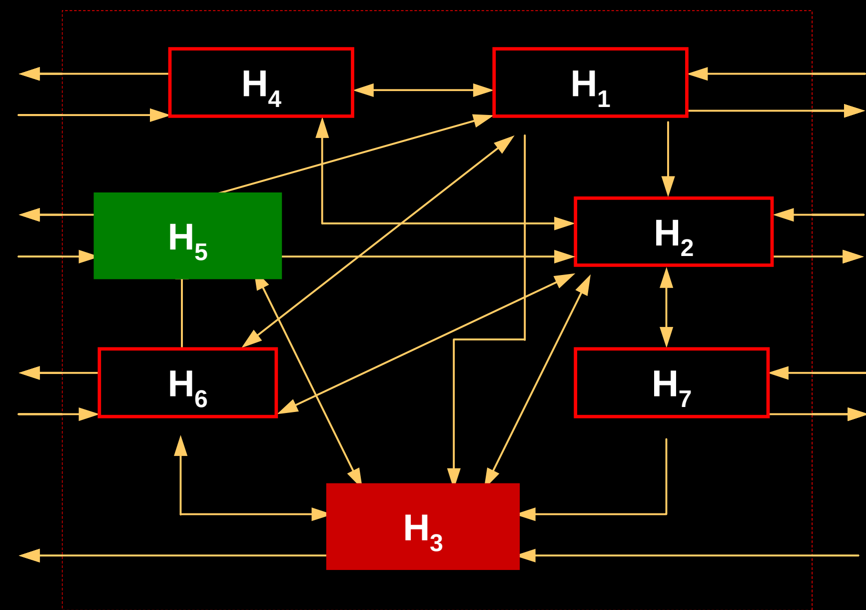
Results (2)

Control Difference, CD

$$CD \times 10 = \begin{bmatrix} 0 & -0.2 & -4.1 & 0.2 & 3.6 & 0.3 & -1.0 \\ 0.2 & 0 & -3.6 & 0.4 & 3.0 & -0.3 & -0.9 \\ 4.1 & 3.6 & 0 & 3.9 & 5.5 & 3.6 & 4.4 \\ -0.2 & -0.4 & -3.9 & 0 & 2.9 & -0.3 & -1.3 \\ -3.6 & -3.0 & -5.5 & -2.9 & 0 & -2.6 & -3.5 \\ -0.3 & 0.3 & -3.6 & 0.3 & 2.6 & 0 & -0.7 \\ 1.0 & 0.9 & -4.4 & 1.3 & 3.5 & 0.7 & 0 \end{bmatrix}$$

System Control, cs_i

$$cs_i \times 10 = \begin{bmatrix} -1.3 \\ -1.2 \\ 25.2 \\ -3.2 \\ -21.2 \\ -1.3 \\ 3.0 \end{bmatrix}$$



Results (3)

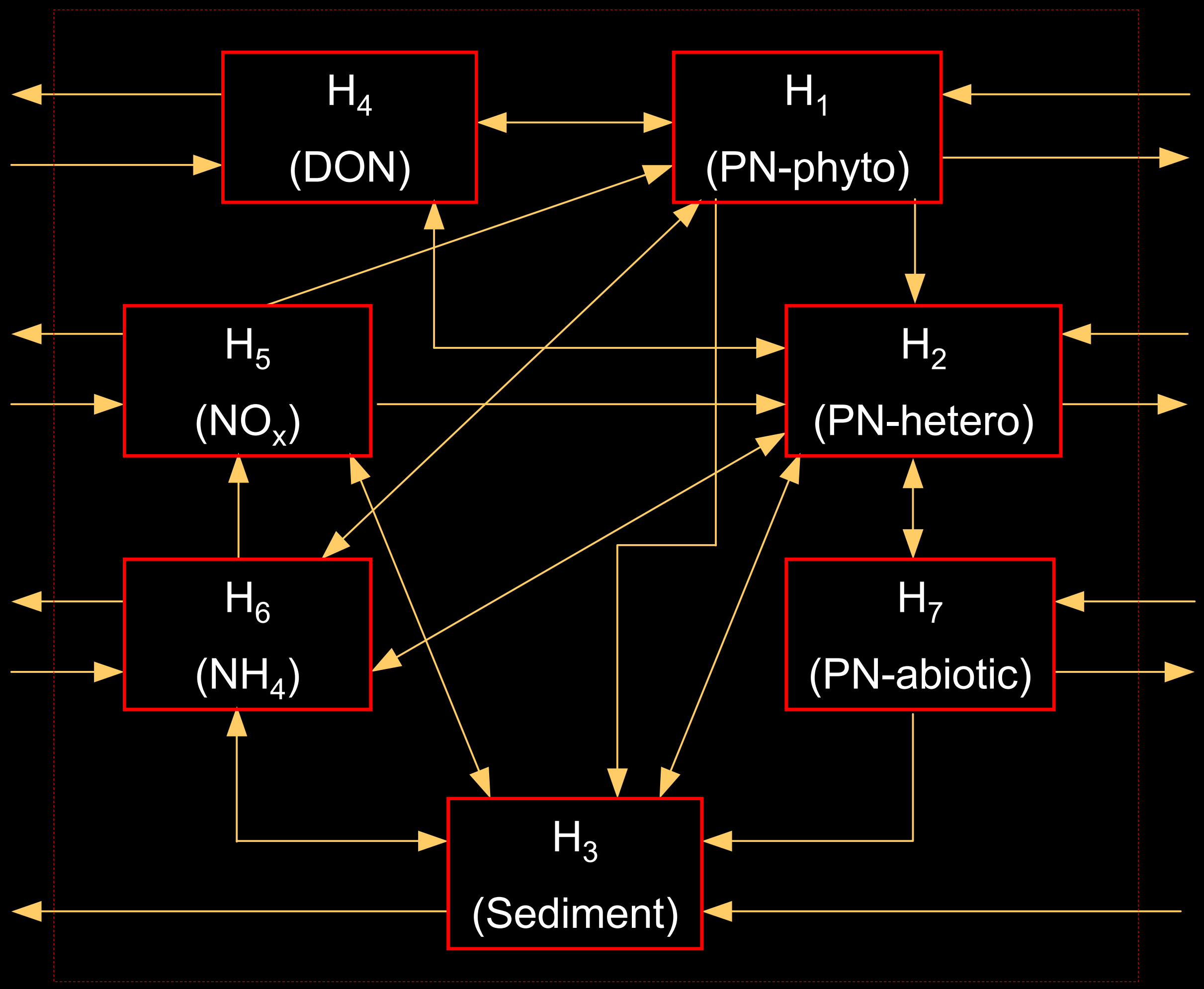
System Control, cs_i

$$cs_i \times 10 = \begin{bmatrix} -1.3 \\ -1.2 \\ 25.2 \\ -3.2 \\ -21.2 \\ -1.3 \\ 3.0 \end{bmatrix}$$

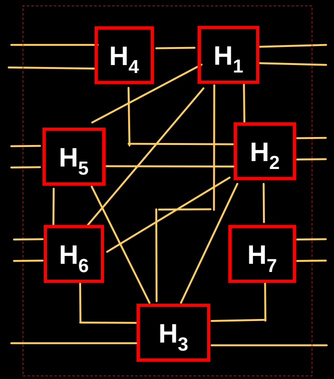
$$\sum_{i=1}^{n} cs_i = 0$$

Nitrogen flow digraph in the Neuse River Estuary

(Christian and Thomas 2003)



Conclusions



- N and N' represent an augmentation or attenuation of system boundary flows, i. e., $T^{out} = Nz$ and $T^{in} = yN'$
- η represents a throughflow normalized and boundary value scaled (z = y = 1) metric of N and N‘
- Comparative metrics CR, CD, and cs offer quantitative values for pair-wise dominance relationships in networks
- Nitrates-Nitrites and Sediment are at opposite ends of the environ control spectrum in the nitrogen model of the Neuse River Estuary: USA
- Currently assessing the influence of throughflow magnitude, component storage, and resident times on results