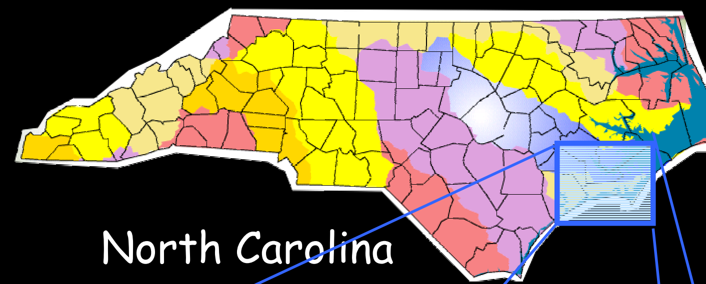
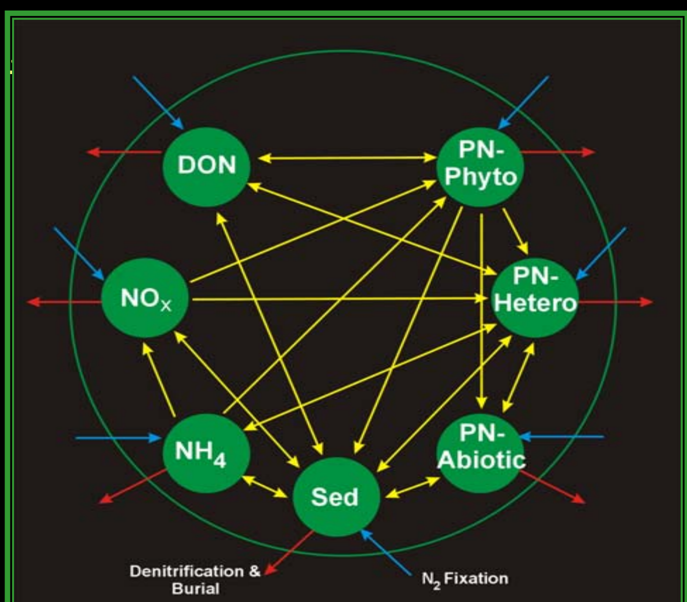
Indirect Effects & Distributed Control in Ecosystems



North Carolina

Environs and Network Environ Analysis: Introduction and Overview



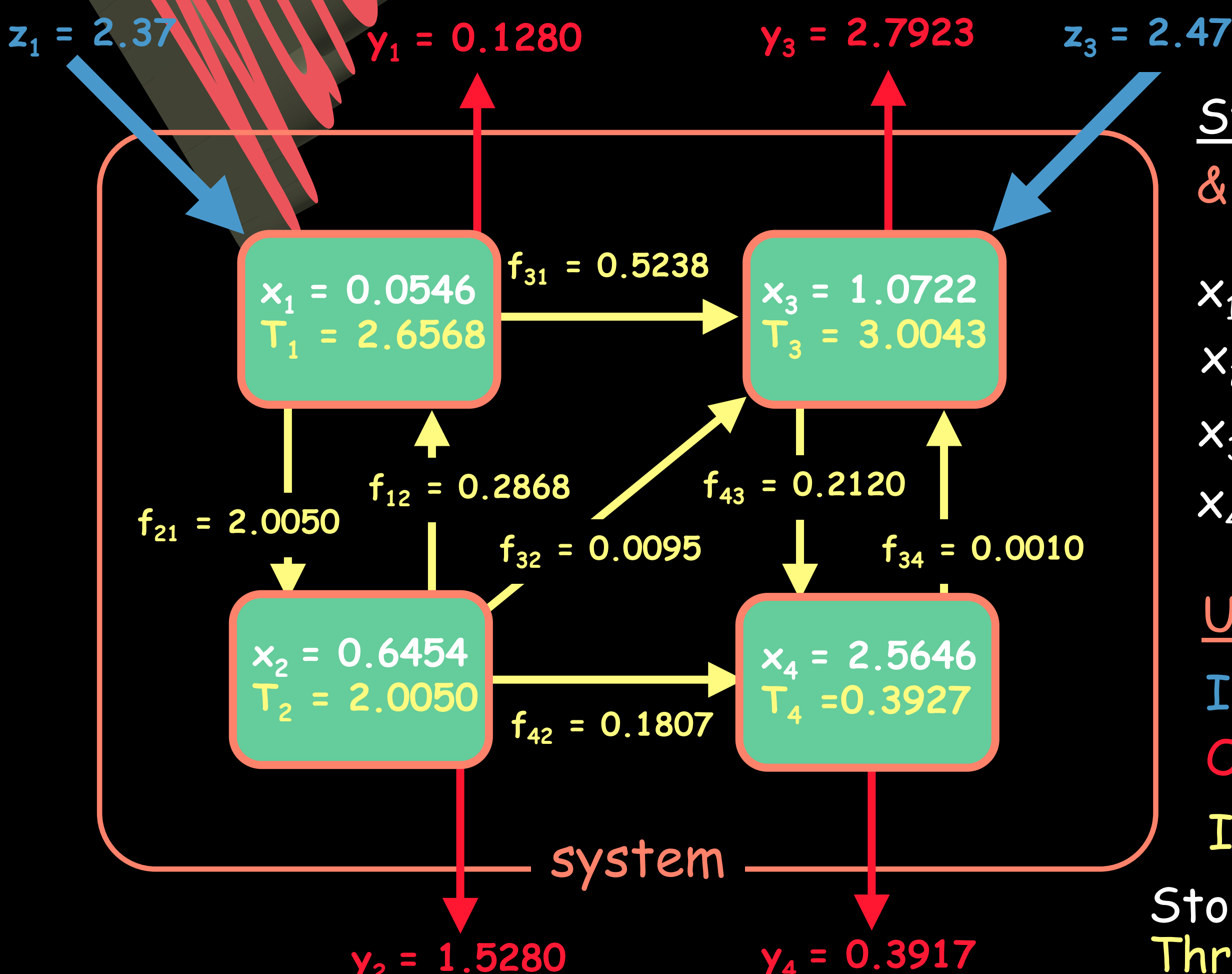
Nitrogen Model
Neuse River Estuary, North Carolina, USA
(Christian and Thomas 2003)

http://www.coe.uncc.edu/~jdbowen/neem/

Water Balance Model

Okefenokee Swamp, Georgia, USA

(B. C. Patten & J. H. Matis 1982. Ecol. Mod. 16: 1-50)



State Variables (x_i)

& Throughflows (T_i)

x_1, T_1 = upland surface water

x_2 , T_2 = upland groundwater

x_3 , T_3 = swamp surface water

x_4 , T_4 = swamp subsurface water

Units (25-year mean values)

Inputs (z_j): 10^9 m^3 H_2O y^{-1}

Outputs (y_i): 10^9 m^3 H_2O y^{-1}

Interflows (f_{ij}): 10^9 m^3 H_2O y^{-1}

Storages (x_i): 10^9 m^3 H_2O

Throughflows (T_i): 10^9 m^3 H_2O y^{-1}

Water Balance Model

Matrix Arrays

$$F = \begin{pmatrix} -T_1 & f_{12} = 0.2868 & 0 & 0 \\ f_{21} = 2.0050 & -T_2 & 0 & 0 \\ f_{31} = 0.5238 & f_{32} = 0.0095 & -T_3 & f_{34} = 0.0010 \\ 0 & f_{42} = 0.1807 & f_{43} = 0.2120 & -T_4 \end{pmatrix}$$

flow matrix

$$z = \begin{pmatrix} z_1 = 2.37 \\ z_2 = 0 \\ z_3 = 2.47 \\ z_4 = 0 \end{pmatrix}$$

input vector

$$y = \begin{pmatrix} y_1 = 0.1280 \\ y_2 = 1.5280 \\ y_3 = 2.7923 \\ y_4 = 0.3917 \end{pmatrix}$$

output vector

$$T = \begin{pmatrix} T_1 = 2.6568 \\ T_2 = 2.0050 \\ T_3 = 3.0043 \\ T_4 = 0.3927 \end{pmatrix}$$

throughflow vector

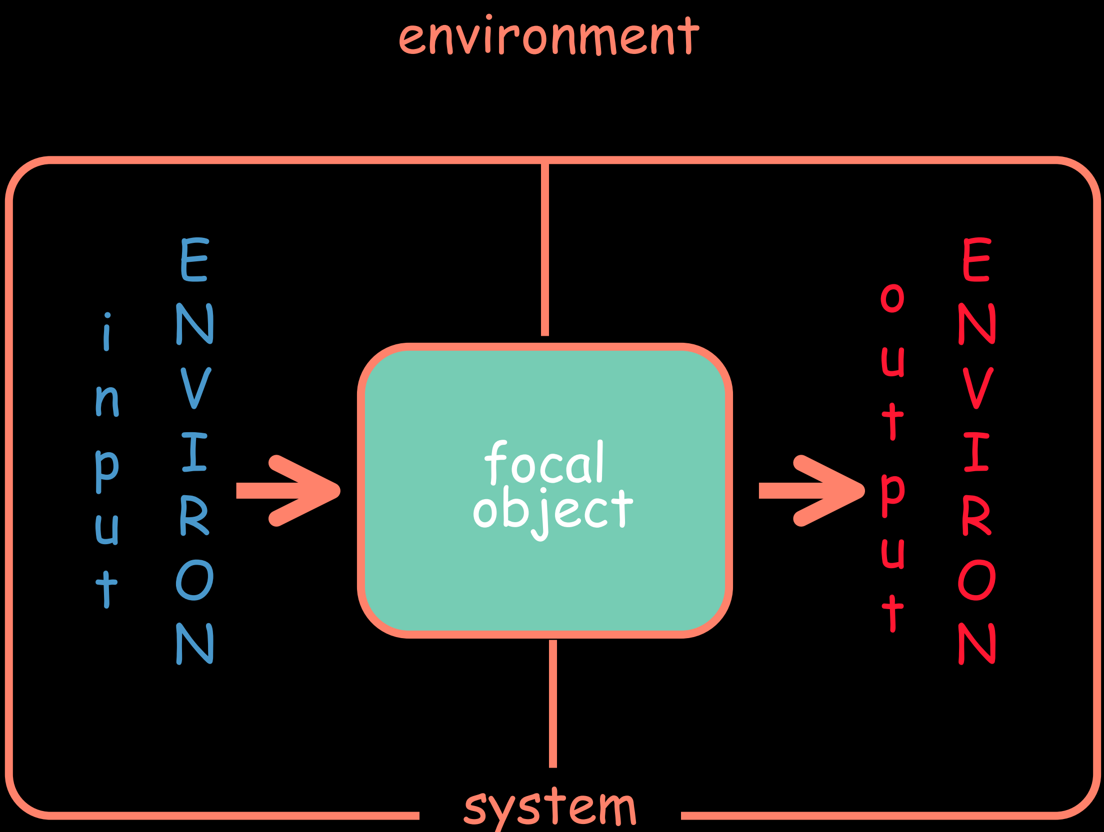
$$x = \begin{pmatrix} x_1 = 0.0546 \\ x_2 = 0.6454 \\ x_3 = 1.0722 \\ x_4 = 2.5646 \end{pmatrix}$$

storage vector

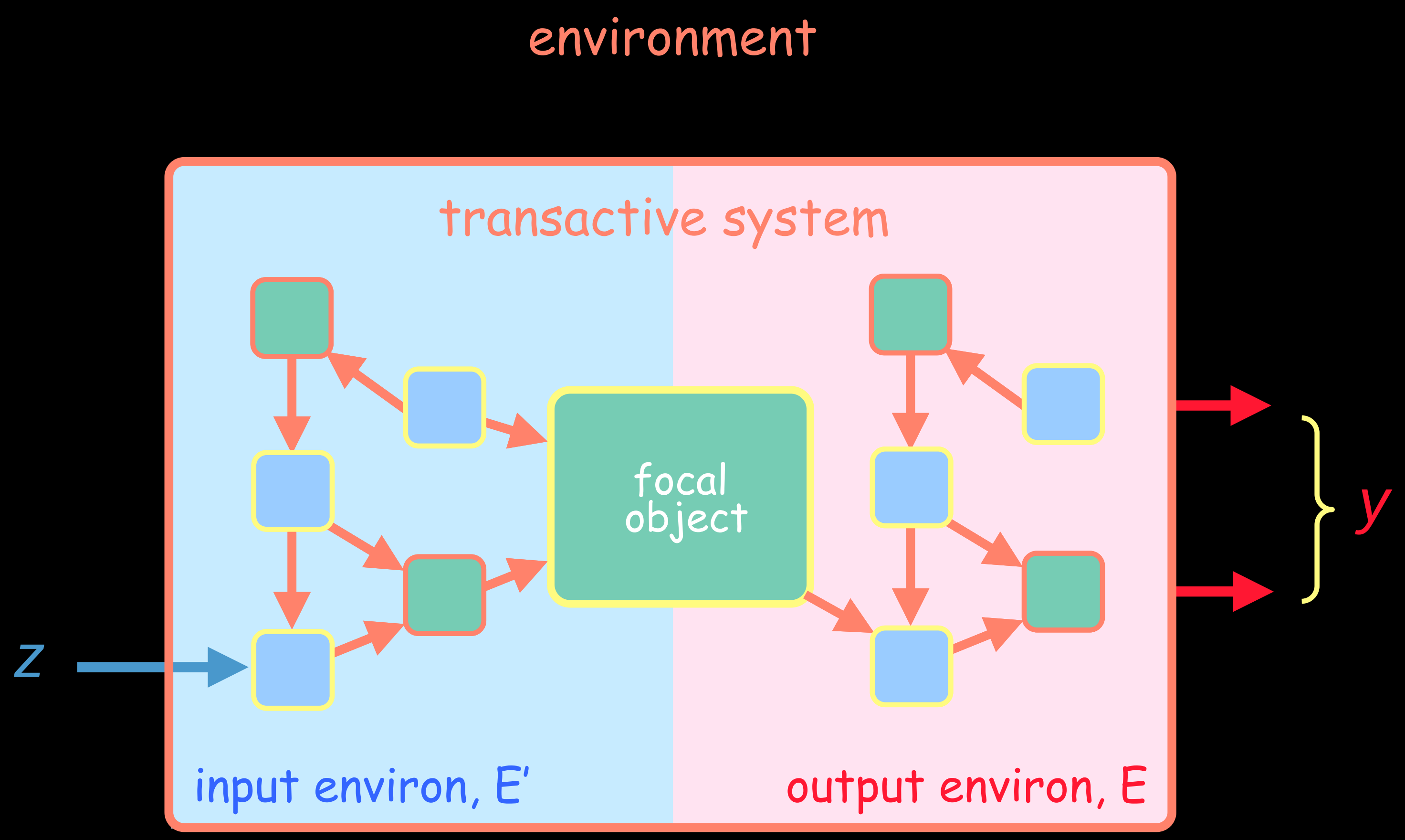
Environs

A formal concept of environment

(B. C. Patten 1978. Ohio J. Sci. 78: 206-222)



Environs as Networks ...
of direct and indirect transactions

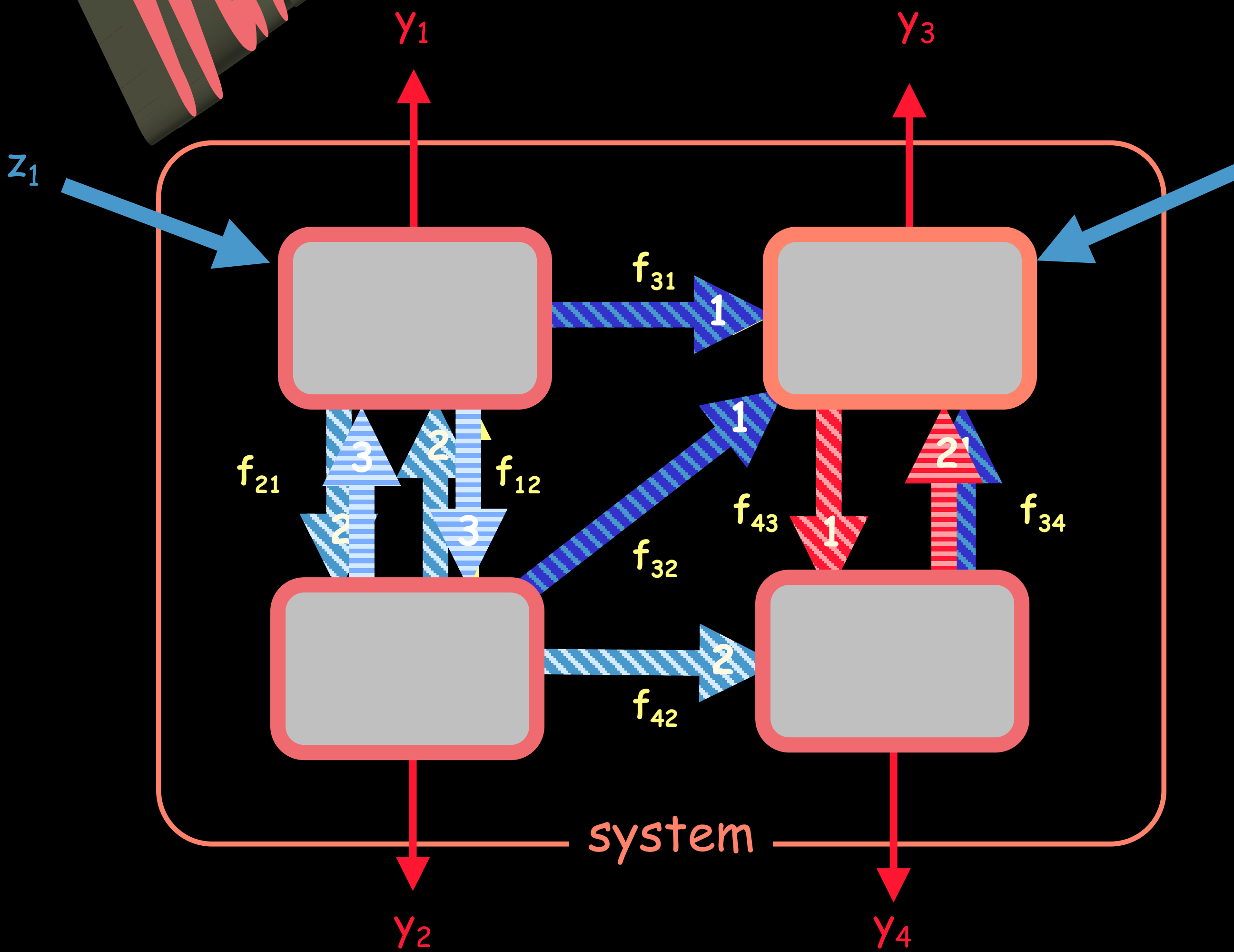


a network of afferent transactions

a network of efferent transactions

Water Model Environs...

(preliminaries)

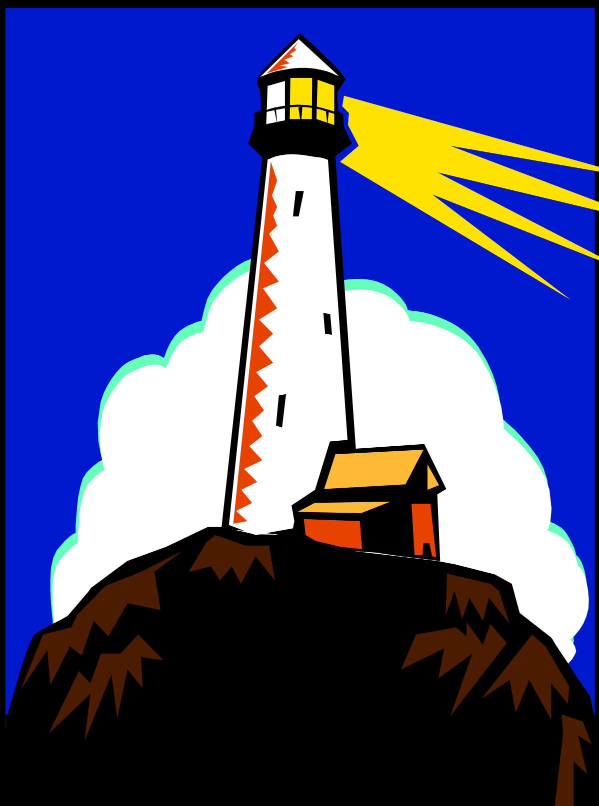


Trace forward through output environs.

Trace backward through input environs.

Every compartment in a system has a pair of such environs.

Mathematical Methods of Environ Analysis



Pathway analysis

Primary methods (3):

Structural analysis …

1. **Pathway analysis**: identifies, counts, and classifies transport pathways in networks.

Functional analyses …

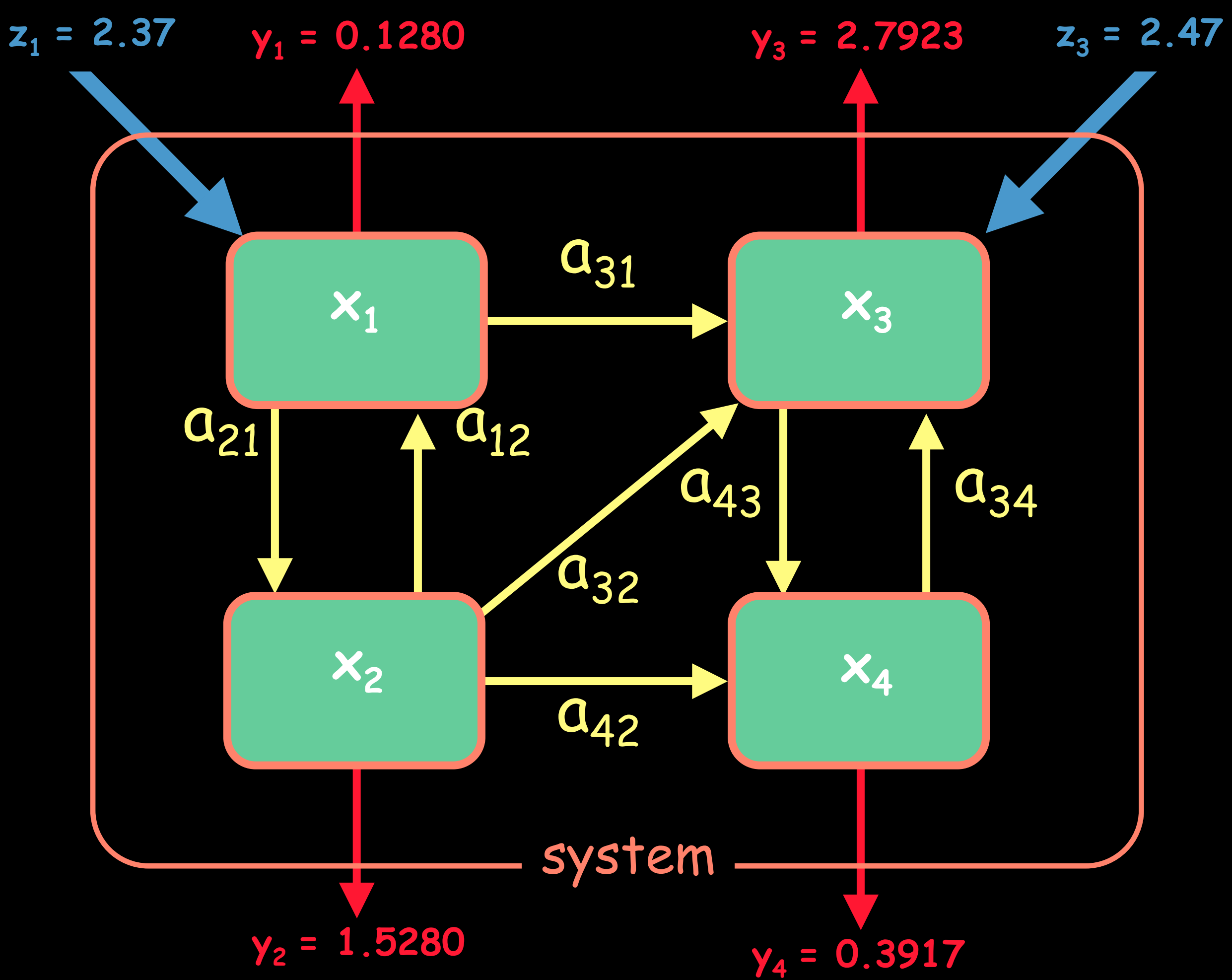
2. **Throughflow analysis**: maps boundary inputs z_j and outputs y_k into interior throughflows, T_i.
3. **Storage analysis**: maps boundary inputs z_j and outputs, y_k into interior storages, T_i.

Secondary methods (2):

4. **Utility analysis**: measures direct and indirect values (benefits and costs) conferred to objects by their participation in networks.
5. **Control analysis**: measures direct and indirect control exerted between each object pair in a system.

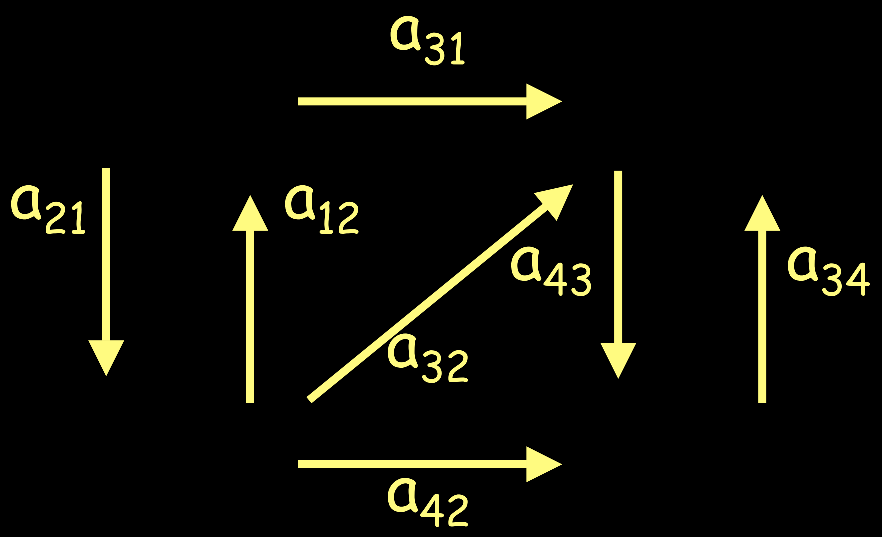
Structural Environ Analysis

Arcs & Adjacency Coefficients



Structural Environ Analysis

Direct Links



Structural Environ Analysis
Adjacency Coefficients

a_{31}

a_{21} a_{12}

a_{43} a_{34}

a_{32}

a_{42}

Structural Environ Analysis

Adjacency Matrix Forming

$$
\begin{array}{cccc}
 & a_{12} & & \\
a_{21} & & & \\
a_{31} & a_{32} & & a_{34} \\
 & a_{42} & a_{43} &
\end{array}
$$

Structural Environ Analysis

Adjacency Matrix

$$A = (a_{ij}) = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

Structural Environ Analysis
Adjacency Matrix

$$A = (a_{ij}) = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} = \begin{pmatrix} a_{11} & 1 & 0 & 0 \\ 1 & a_{22} & 0 & 0 \\ 1 & 1 & a_{33} & 1 \\ 0 & 1 & 1 & a_{44} \end{pmatrix}$$

$$a_{ij} = \begin{cases} 1 \text{ if direct arc } (\longrightarrow) \text{ connects } j \text{ to } i \\ 0 \text{ otherwise} \end{cases}$$

$$a_{ii} = \begin{cases} 0 \text{ for throughflow analysis} \\ 1 \text{ for storage analysis} \end{cases}$$

Structural Environ Analysis

Adjacency Matrix Powers

1. In connected networks pathways grow exponentially in number with increasing length, m.

2. The number of pathways of length m reaching i from j is given by the elements $a_{ij}^{(m)}$ of A^m.

3. The number of time-forward pathways of all lengths $0 \leq m < \infty$ is given by a <u>divergent</u> adjacency matrix power series:

4. The number of time-backard pathways of all lengths $0 \leq m < \infty$ is given by a transposed adjacency matrix power series:

$$I + A + A^2 + A^3 + \dots + A^m + \dots$$

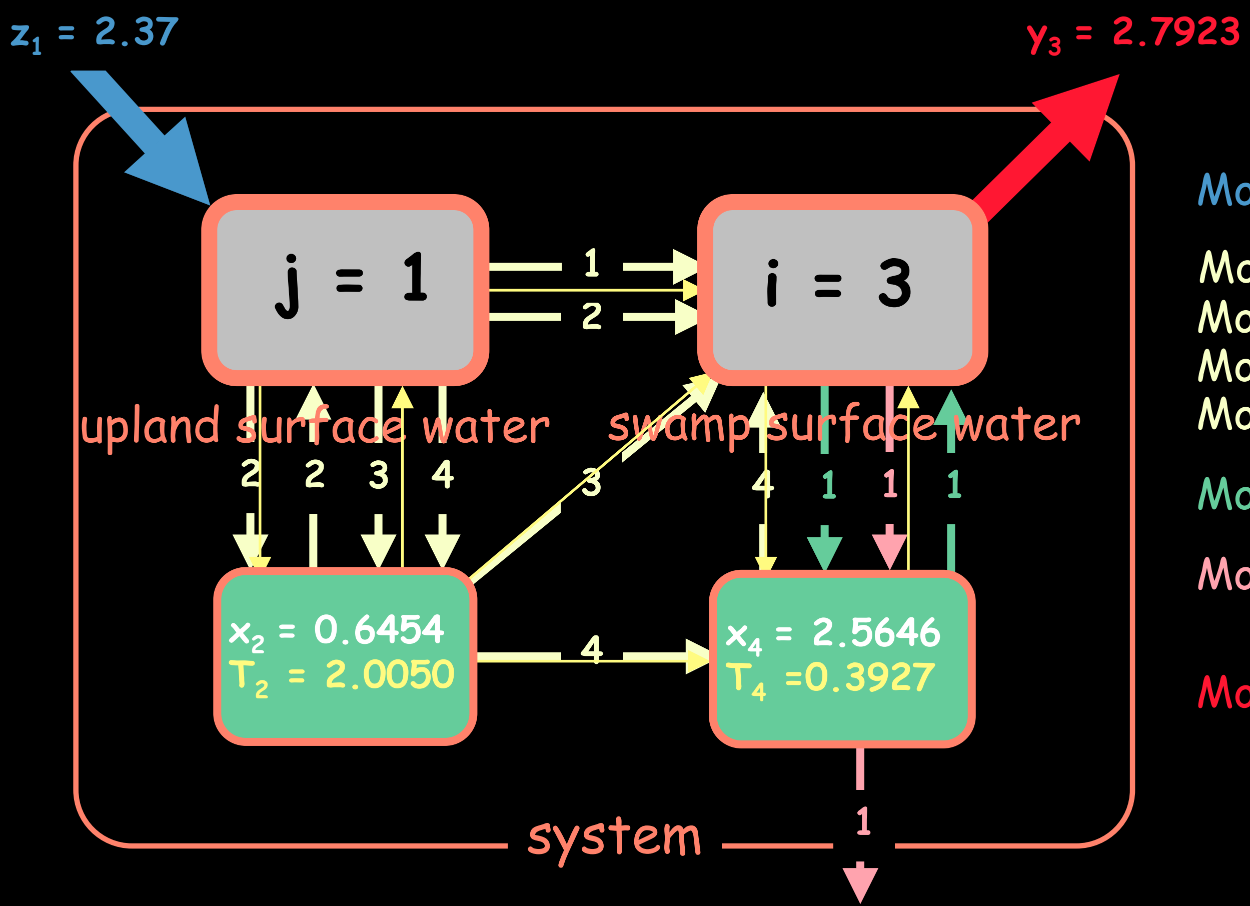
$$I + A^T + \underbrace{(A^T)^2 + (A^T)^3 + \dots + (A^T)^m}_{\text{indirect pathways } (m > 1)} + \dots$$

boundary (m = 0)

direct arc (m = 1)

Structural Environ Analysis

Five Pathway Modes (Water Model)



Mode 0: boundary input

Mode 1.1
Mode 1.2
Mode 1.3
Mode 1.4
} first passage

Mode 2: terminal node cycling

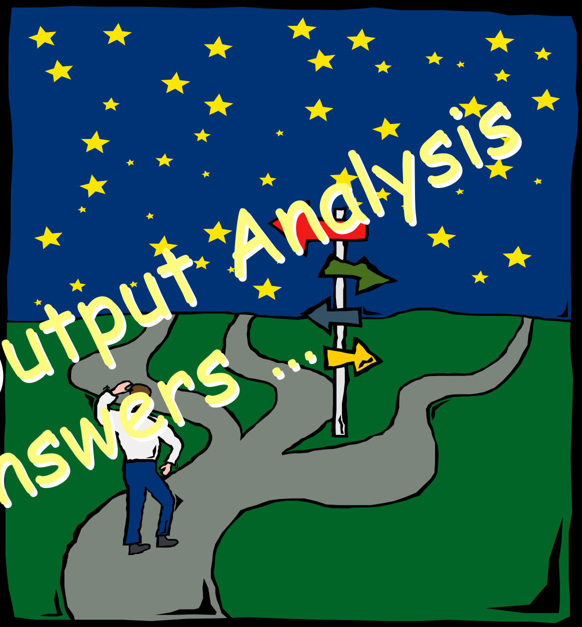
Mode 3: indirect dissipation via system interior

Mode 4: direct boundary dissipation

Functional Environ Analysis
Basic Questions

Q1. How can direct and indirect effects propagated by transactions between all originating & terminating node pairs in networks of arbitrary, large-number (exponentiating), pathway structure be measured?

(This requires accounting for transport over all direct and indirect pathways to achieve the graph-theoretic property, transitive closure.)

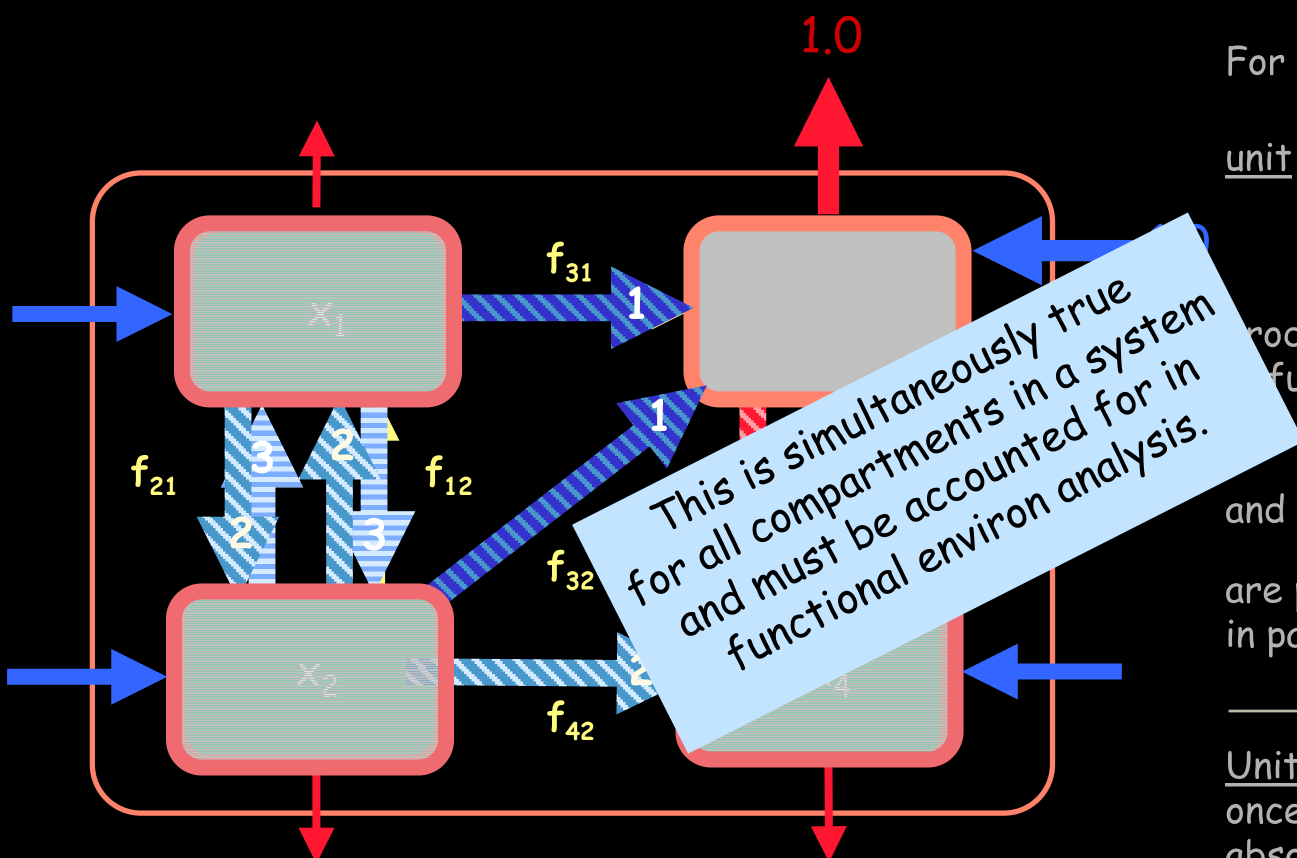


Variations on Input-Output Analysis provide the answers …

Q2. How can such measured effects be then partitioned and assigned to individual environs?

Functional Environ Analysis

Basic Approach—Unit Environs



For each compartment in a model …

unit inputs …

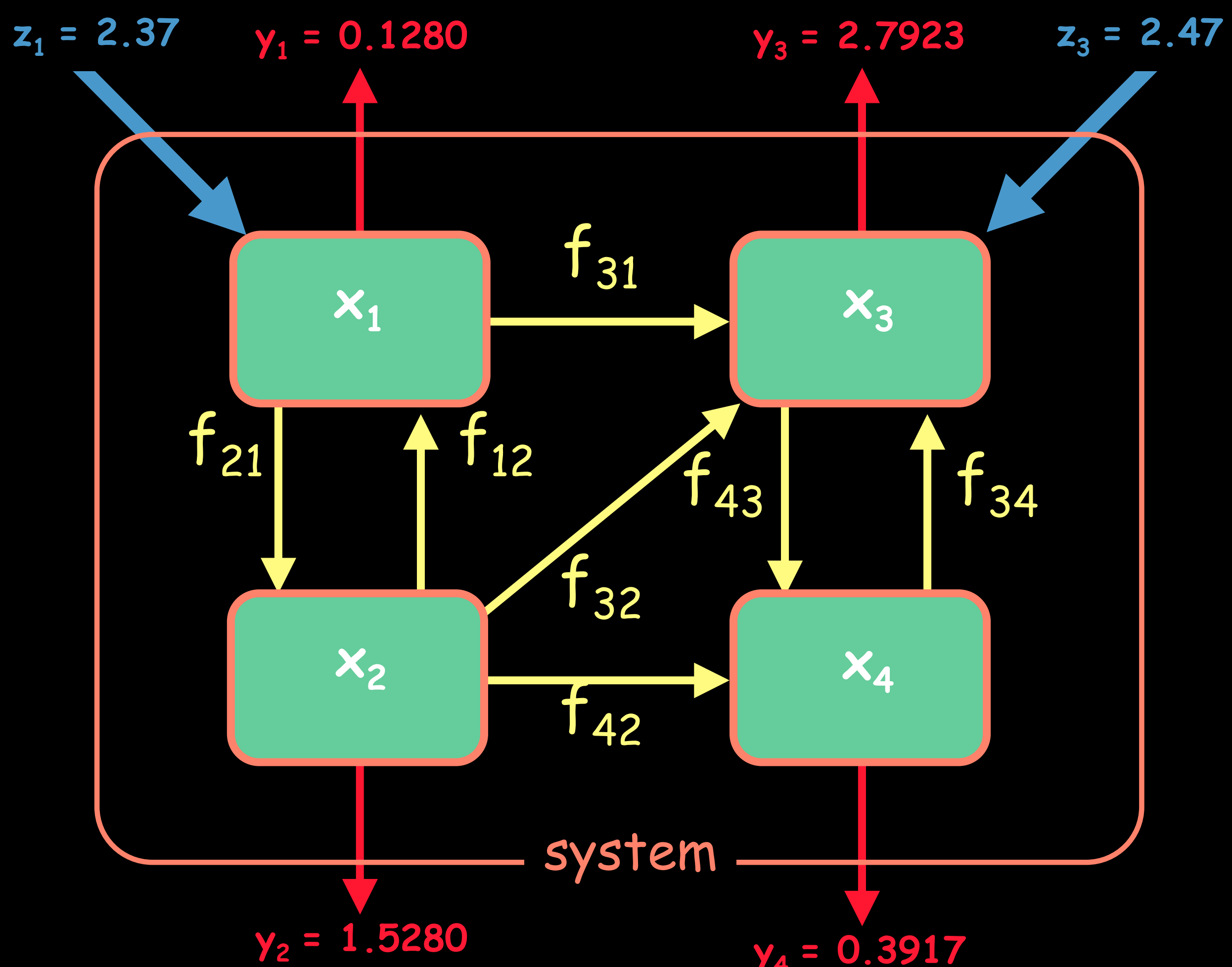
roduce unit output environs
future time …

and unit outputs …

are produced by unit input environs
in past time.

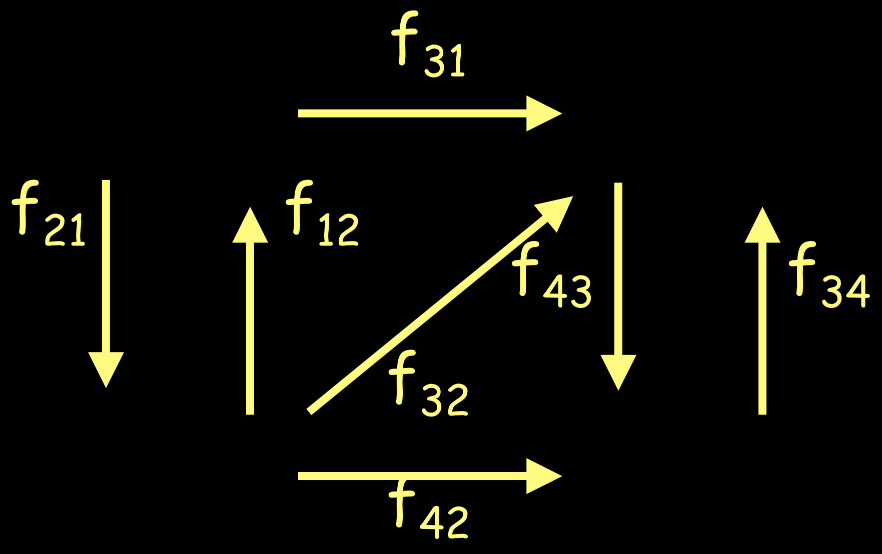
Unit environs are partition elements; once calculated, they are scaled to absolute numerical values by their respective z and y values.

Functional Environ Analysis
Flow Digraph—Time Continuous



Functional Environ Analysis

Flow Arcs and Flow Functions



Functional Environ Analysis
Flow Functions

f_{31}

f_{21} f_{12}

f_{43} f_{34}

f_{32}

f_{42}

Functional Environ Analysis

Flow Functions

f_{12}

f_{21}

f_{31} f_{32} f_{34}

f_{42} f_{43}

Functional Environ Analysis

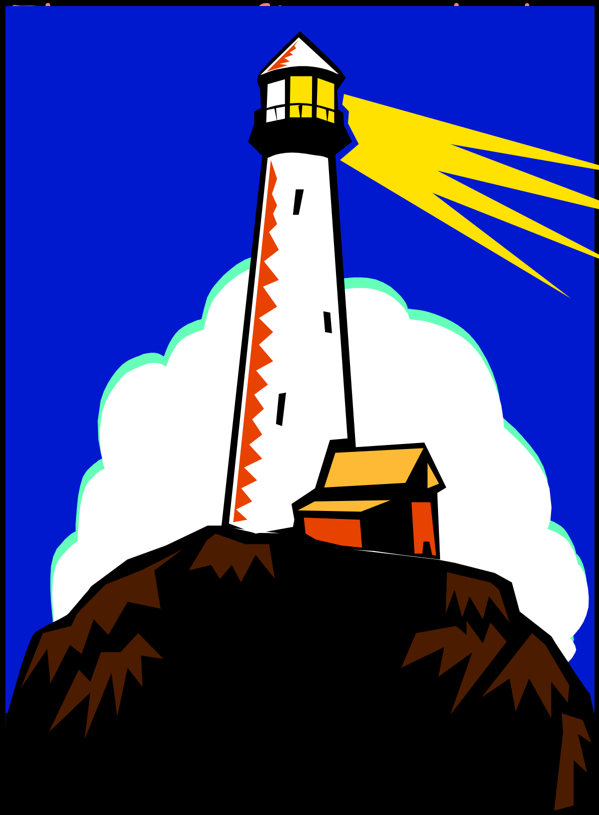
Flow Matrix

$$F = (f_{ij}) = \begin{pmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{pmatrix}$$

Functional Environ Analysis

Mathematical Methods

Throughflow analysis



Primary methods (3):

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Secondary methods (2):

4. **Utility analysis**: measures direct and indirect values (benefits and costs) conferred to objects by their participation in networks.
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Throughflow Environ Analysis

Flow Matrices in State Transition Function Derivatives

$$F = (f_{ij}) = \begin{pmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{pmatrix} = \begin{pmatrix} -T_1 & f_{12} & f_{13} & f_{14} \\ f_{21} & -T_2 & f_{23} & f_{24} \\ f_{31} & f_{32} & -T_3 & f_{34} \\ f_{41} & f_{42} & f_{43} & -T_4 \end{pmatrix}$$

alternative state transition functions (derivatives):

$$\phi': dx/dt = F\,\mathbf{1} + z = -F^T\mathbf{1} - y$$

output environ analysis (prospective)

input environ analysis (retrospective)

Throughflow Environ Analysis

Problem Statement

At steady state: ϕ': $dx/dt = 0$

$$F1 + z = -F^T1 - y = 0 \;\ldots$$

For output environ analysis, find a matrix N that in future time maps boundary input z into interior throughflow:

$$T = N\,z$$

Variation on the Leontief Theme
(driver oriented)

For input environ analysis, find a matrix N ' that in past time maps boundary output y into interior throughflow:

$$T = y\,N'$$

Leontief Input–Output Analysis
(demand oriented)

Throughflow Environ Analysis

N and N' Mapping Matrices

<u>output environ analysis</u>

$$dx/dt = F1 + z = [f_{ij}]\,1 + z$$

Nondimensionalization:

$$\begin{cases} g_{ij} = f_{ij}/T_j \\ g_{jj} = f_{jj}/T_j = -T_j/T_j = -1 \end{cases}$$

$$G = [g_{ij}]$$

$$dx/dt = GT + z$$

At steady state ($dx/dt = 0$):

$$\boxed{T = (-G)^{-1} z = N z}$$

<u>input environ analysis</u>

$$dx/dt = -F^T 1 - y = -[f_{ij}]\,1 - y$$

$$\begin{cases} g_{ij}' = f_{ij}/T_i \\ g_{ii}' = f_{ii}/T_i = -T_i/T_i = -1 \end{cases}$$

$$G' = [g_{ij}']$$

$$dx/dt = -G'T - y$$

$$\boxed{T = y\,(-G')^{-1} = y\,N'}$$

Throughflow Environ Analysis
Power Series Decomposition

Following pathway analyses:

$$I + A + A^2 + A^3 + \ldots + A^m + \ldots$$

$$I + A^T + \underbrace{(A^T)^2 + (A^T)^3 + \ldots + (A^T)^m + \ldots}_{\text{indirect pathways } (m > 1)}$$

boundary ($m = 0$)

direct arc ($m = 1$)

indirect pathways ($m > 1$)

N and N′ have the following series decompositions:

$$N = I + G + G^2 + G^3 + \ldots + G^m + \ldots$$

$$N' = I + G' + G'^2 + G'^3 + \ldots + G'^m + \ldots$$

Throughflow Environ Analysis

Power Series Decomposition

... showing boundary, direct, and indirect interior flow contributions to throughflows:

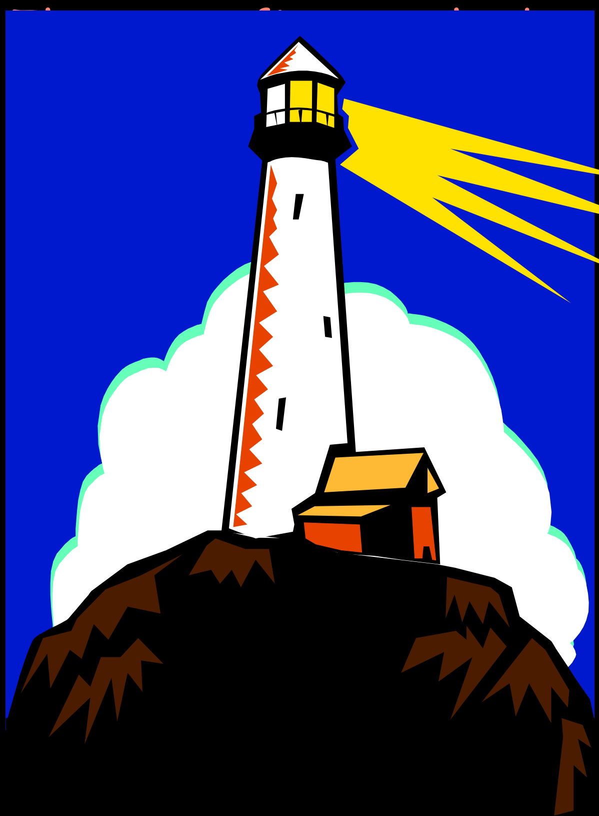
$$T = (I + G + G^2 + G^3 + \ldots + G^m + \ldots)\, z$$

$$T = \underbrace{y \left(\underbrace{I}_{\substack{\text{boundary} \\ (m = 0)}} + \underbrace{G'}_{\substack{\text{direct} \\ (m = 1)}} + \underbrace{G'^2 + G'^3 + \ldots + G'^m + \ldots}_{\substack{\text{indirect} \\ (m > 1)}} \right)}_{\text{empirical flows}}$$

Functional Environ Analysis

Mathematical Methods

Storage analysis



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Storage Environ Analysis
Problem Statement

At steady state: ϕ': $dx/dt = 0$

$$F1 + z = -F^T1 - y = 0 \ldots$$

For output environ analysis, find a matrix S that in future time maps boundary input z into interior storage:

$$x = S\, z$$

For input environ analysis, find a matrix S ′ that in past time maps boundary output y into interior storage:

$$x = y\, S'$$

Storage Environ Analysis
Standard Form Linear System—A Problem

State transition function derivative, flow matrix form:

$$\phi': dx/dt = F\mathbf{1} + z = -F^{T}\mathbf{1} - y$$

Standard linear form:

$$\phi': dx/dt = Cx + z = -C'x - y$$

Problem:

C and C' have dimensions time^{-1}, which precludes constructing power series

$$x = (I + C + C^2 + C^3 + \ldots + C^m + \ldots)\, z$$

$$x = y\,(I + C' + C'^2 + C'^3 + \ldots + C'^m + \ldots),$$

to account for all orders of flow contributions to storage.

To do this, a **nonstandard form** is required for the state equation ...

Storage Environ Analysis

Markovian Non-standard Form State Equation

Matrix inverses of C and C' do enable mapping boundary inputs or outputs to storages (J. H. Matis & B. C. Patten 1981. Bull. Int. Stat. Inst. 48: 527-565).

$$\boxed{x = (-C)^{-1} z = Sz} \qquad \boxed{x = y(-C')^{-1} = yS'}$$

Nondimensionalizing C and C':

Define: $P = (p_{ij}) = I + C\Delta t$ $\qquad$ $P' = (p_{ij}') = I + C\Delta t$

where: $0 \leq (p_{ij} = c_{ij}\Delta t, i \neq$ … $= 1 + c_{jj}\Delta t) \leq 1$

$0 \leq (p_{ij}' = c_{ij}'\Delta t$ … $, p_{jj}' = 1 + c_{jj}\Delta t) \leq 1$

$c_{jj} = c_j$ … $= -$ … h turnover rate = time constant).

Δt sel … such that, for all i, j, $0 \leq p_{ij}, p_{ij}' \leq 1$

and: P and … Markov chain matrices (probabilistic)

forget all that,

and remember...

Markovian state transition equation derivatives:

$$\phi': dx/dt = (P - I)/\Delta t\, x + z = -(P - I)/\Delta t\, x - y$$

Storage Environ Analysis

Boundary to Storage Mappings

At steady state:

ϕ': $dx/dt = 0$

$((P - I)/\Delta t)x + z = 0$

$-((P' - I)/\Delta t)\, x - y = 0$

$$\boxed{\begin{aligned} x &= (I - P)^{-1}\, z\Delta t \\ &= (Q)z\Delta t = (Q\Delta t)z = Sz \end{aligned}}$$

$$\boxed{\begin{aligned} x &= y\Delta t(I - P')^{-1} \\ &= y\Delta t(Q') = y(Q'\Delta t) = yS \end{aligned}}$$

Power series decomposition:

$$x = (I - P)^{-1}\, z\Delta t = (I + P + P^2 + P^3 + \ldots + P^m + \ldots)\, z\Delta t$$

$$x = y\Delta t\, (I - P')^{-1} = y\Delta t\, (I + P' + P'^2 + P'^3 + \ldots + P'^m + \ldots) \ldots$$

Storage Environ Analysis
Power Series Decomposition

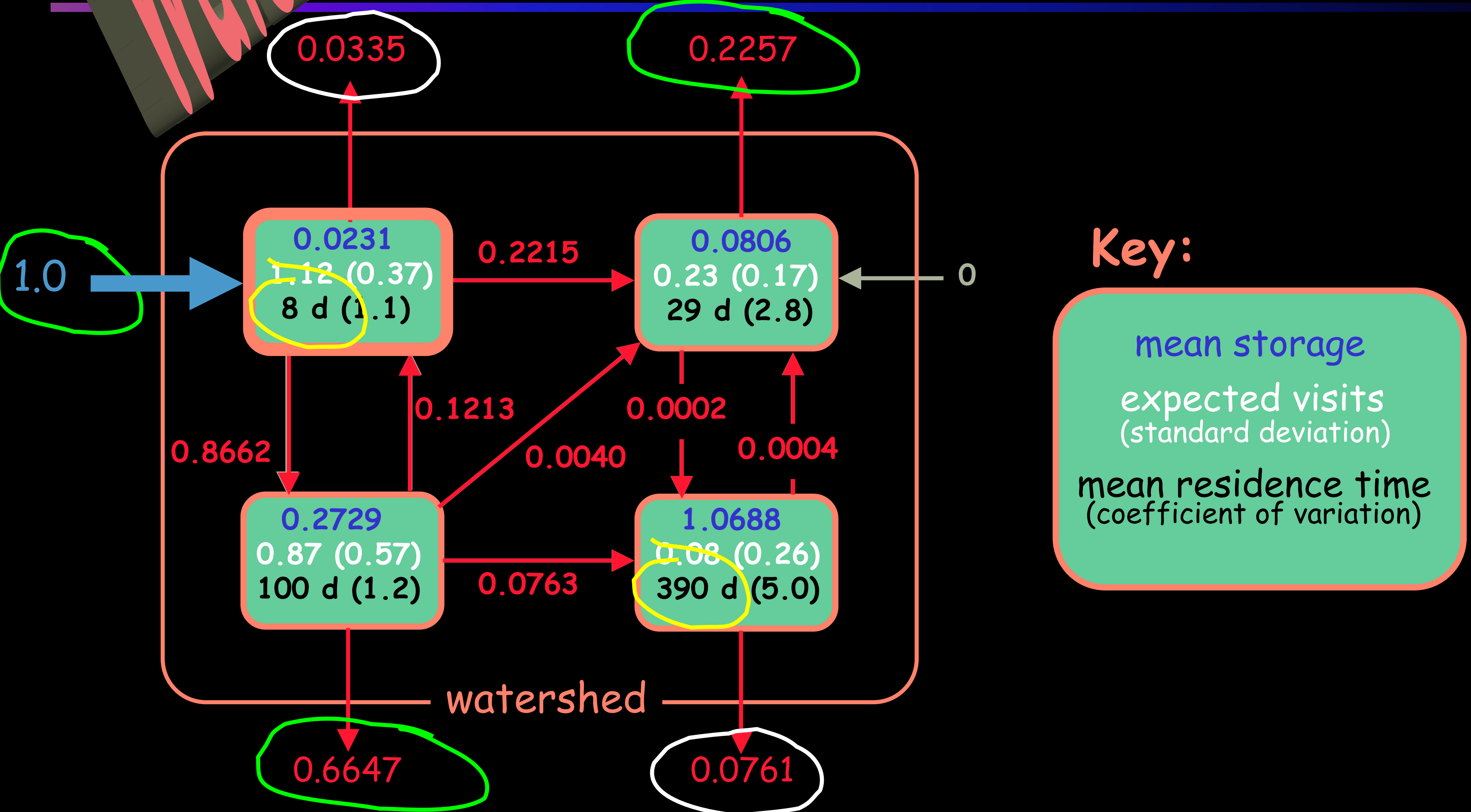
... showing boundary, direct, and indirect interior flow contributions to storages:

$$x = (I + P + P^2 + P^3 + \ldots + P^m + \ldots)\, z\Delta t$$

$$x = \underbrace{y\Delta t\,(\underset{\substack{\text{boundary}\\(m=0)}}{I} + \underset{\substack{\text{direct}\\(m=1)}}{P'} + \underbrace{P'^2 + P'^3 + \ldots + P'^m + \ldots}_{\substack{\text{indirect}\\(m>1)}})}_{\text{empirical stock components}}$$

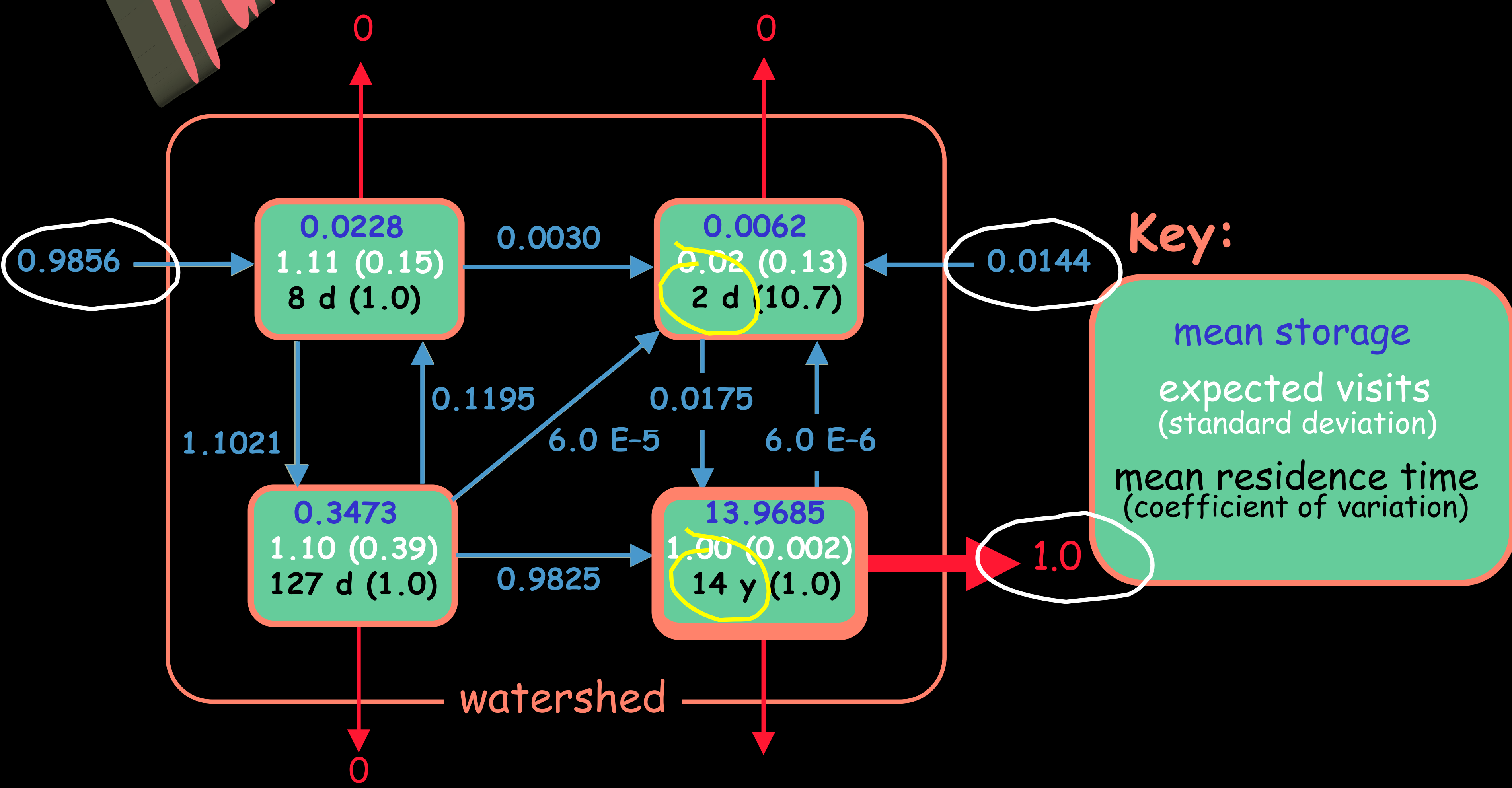
Water Model Environs...

Upland Surface Water Unit Output Environ



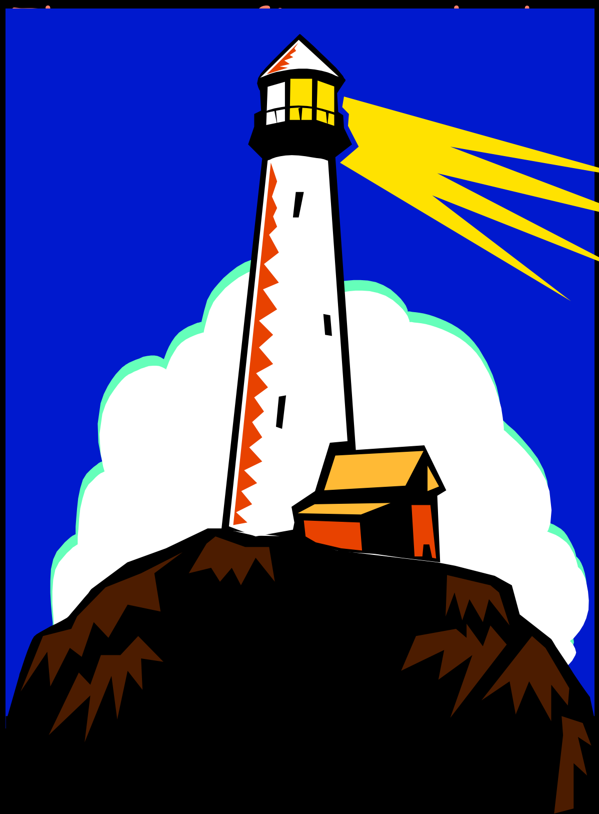
Water Model Environs...

Swamp Subsurface Water Unit Input Environ



Functional Environ Analysis
Mathematical Methods

Control analysis



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Indirect Effects & Distributed Control in Ecosystems

Mathematical Basis

Pathways

$I + A +$

$I + A^T +$

higher order terms

Throughflows

$T = (I + G +$ z

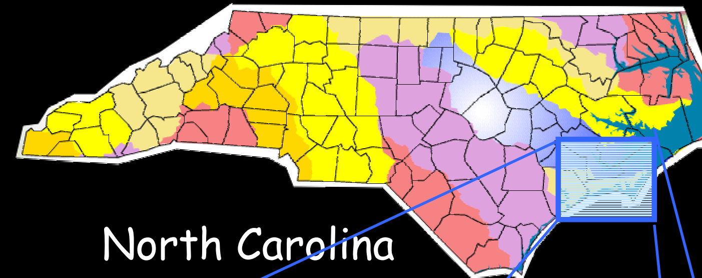
$T = y\,(I + G' +$

Storages

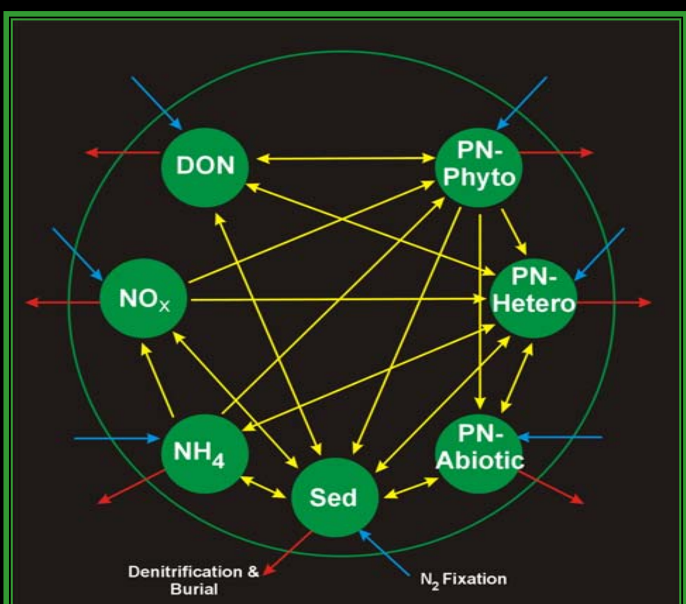
$x = (I + P +$ $z\Delta t$

$x = y\Delta t\,(I + P' +$

Indirect Effects & Distributed Control in Ecosystems



Presentations

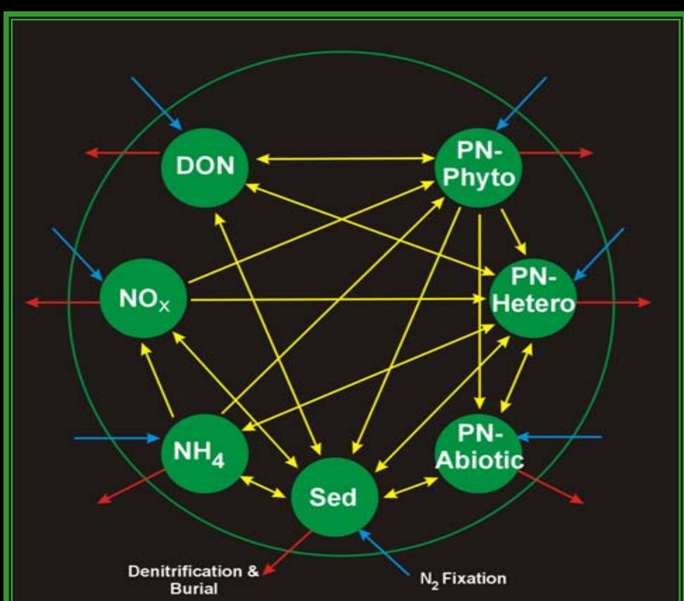


Neuse River Estuary Nitrogen Model

Estuary, USA: static analysis

Indirect Effects & Distributed Control in Ecosystems

Posters



Neuse River Nitrogen Model

Indirect effects—static

D.K. Ga… …t al. (Paper #2)

Network … …sis of a seven-compartment model of nitroge… …euse River Estuary, USA: static analysis

Indirect effects—dynamic

S.J. Wh… …et al. (Paper #4)

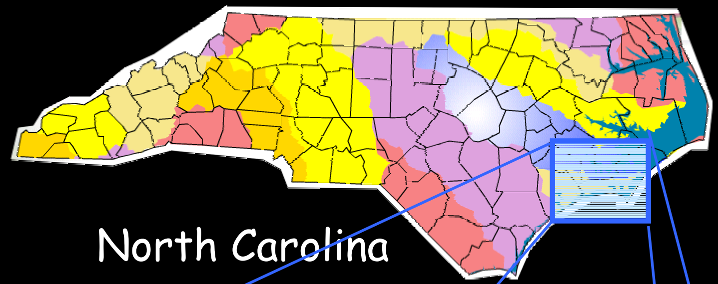
Comparati… …sis of a seven-compartment model of nitroge… …euse River Estuary, USA: time series analysis

Distributed control—dynamic

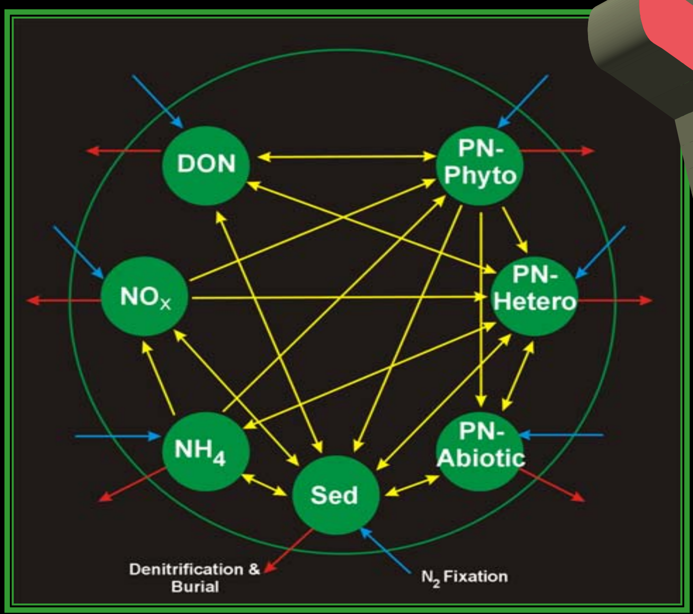
J.R. Sc… …S. Bata, et al. (Paper #6)

Distributed con… …on networks of a seven-compartment model of ni… …the Neuse River Estuary, USA: time series analys…

Thank you



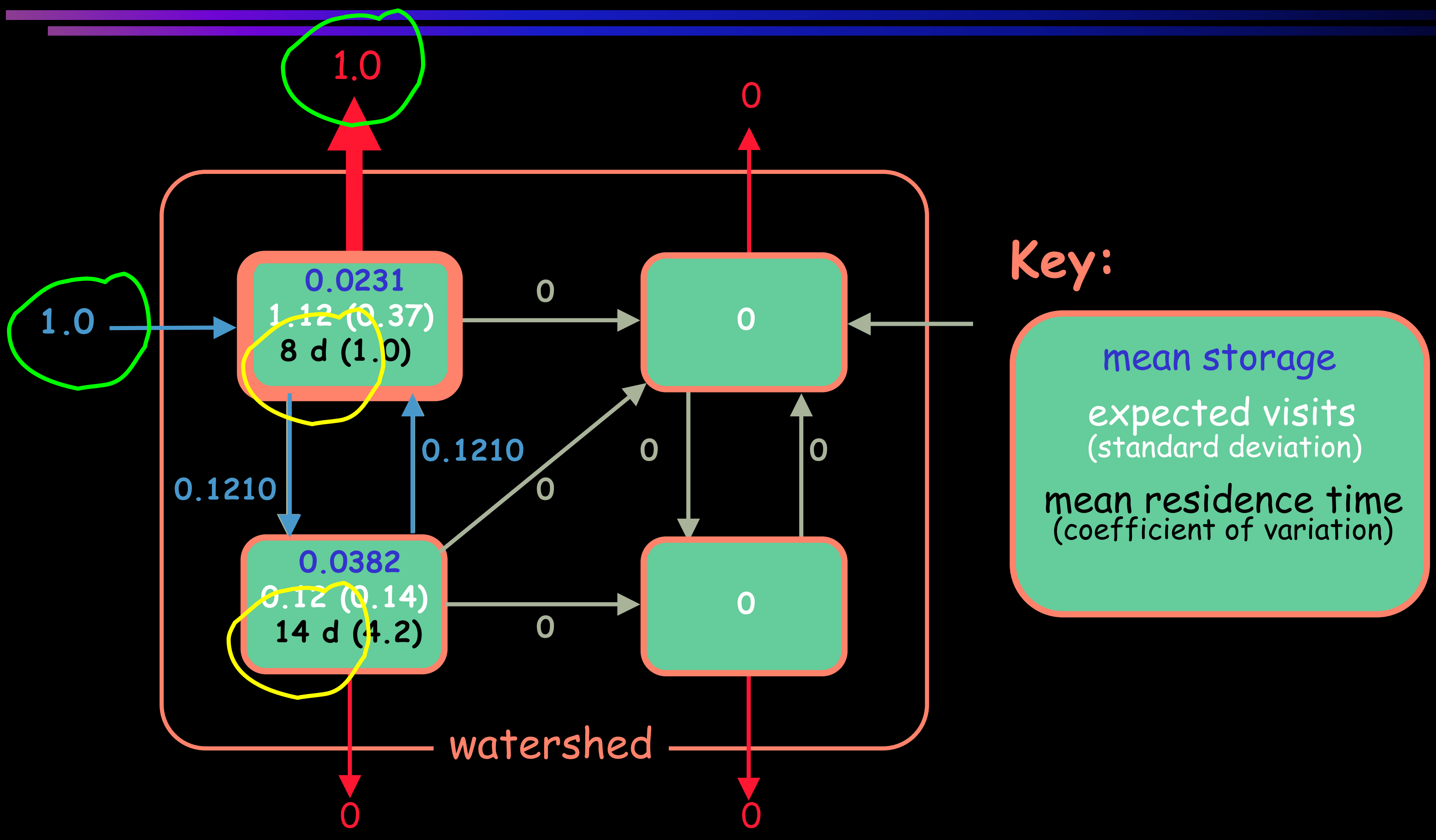
North Carolina



Neuse River Estuary Nitrogen Model

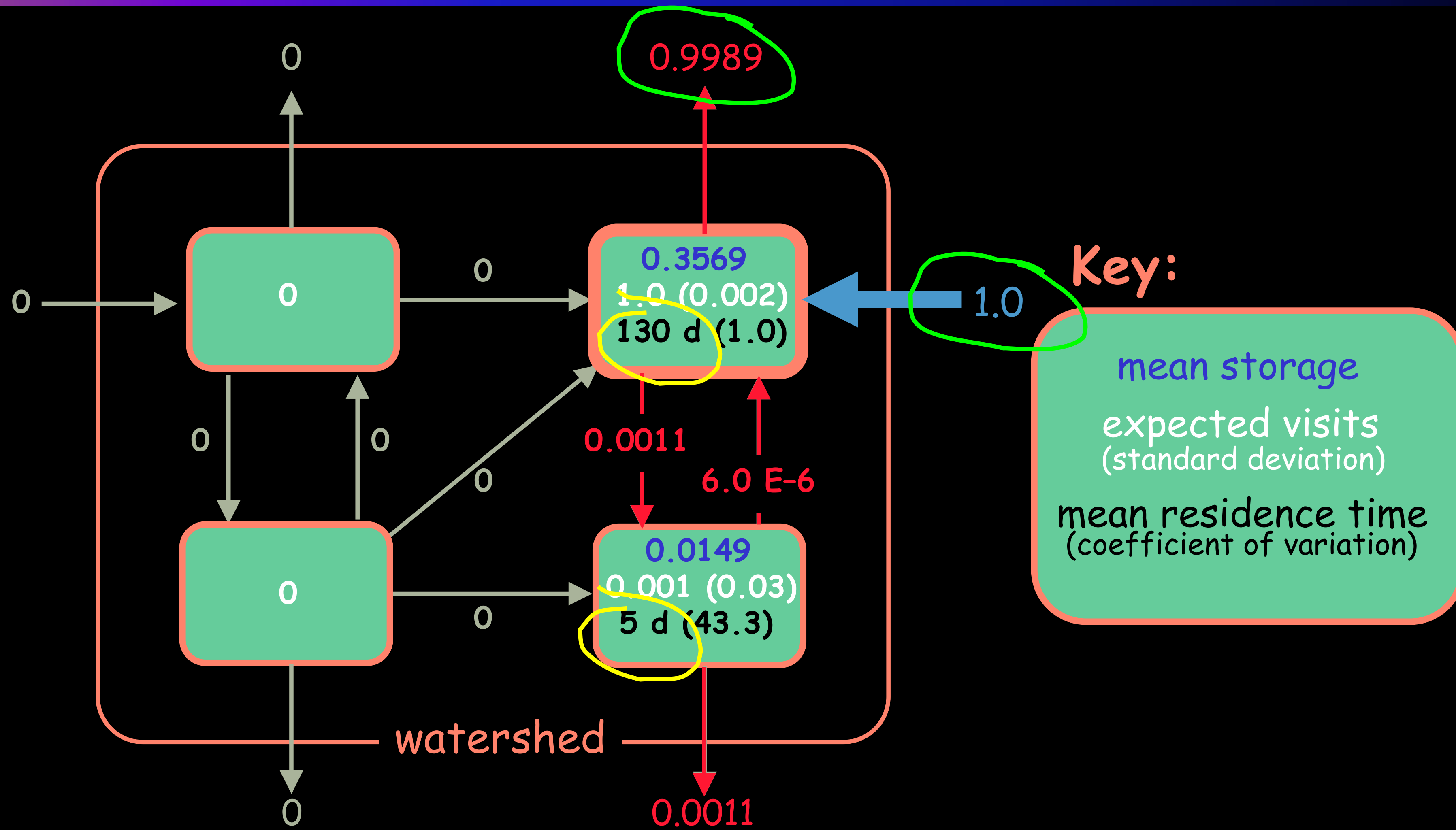
Okekenokee Water Model

Unit Input Environ, Upland Surface Water



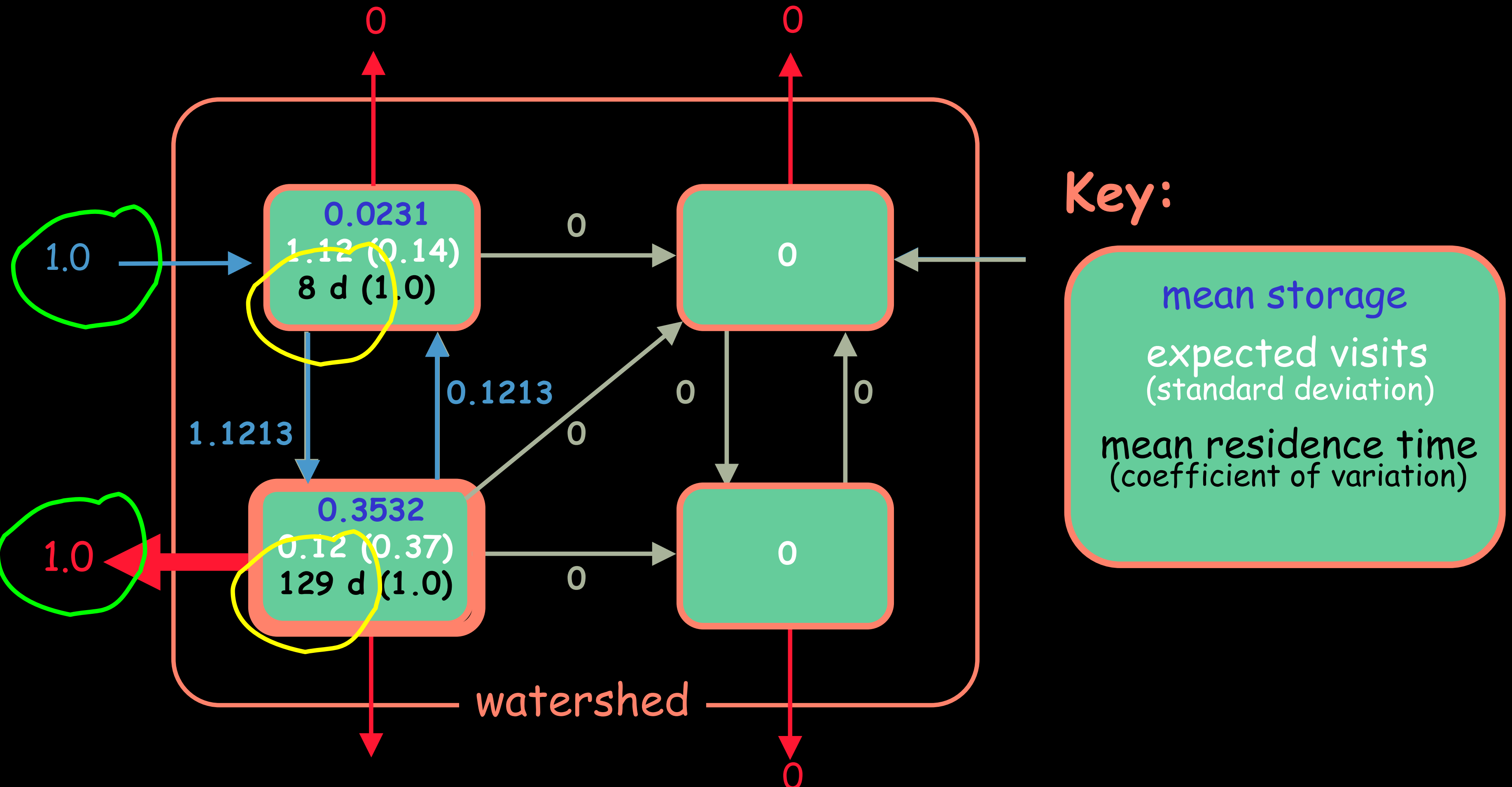
Okekenokee Water Model

Unit Output Environ: Swamp Surface Water



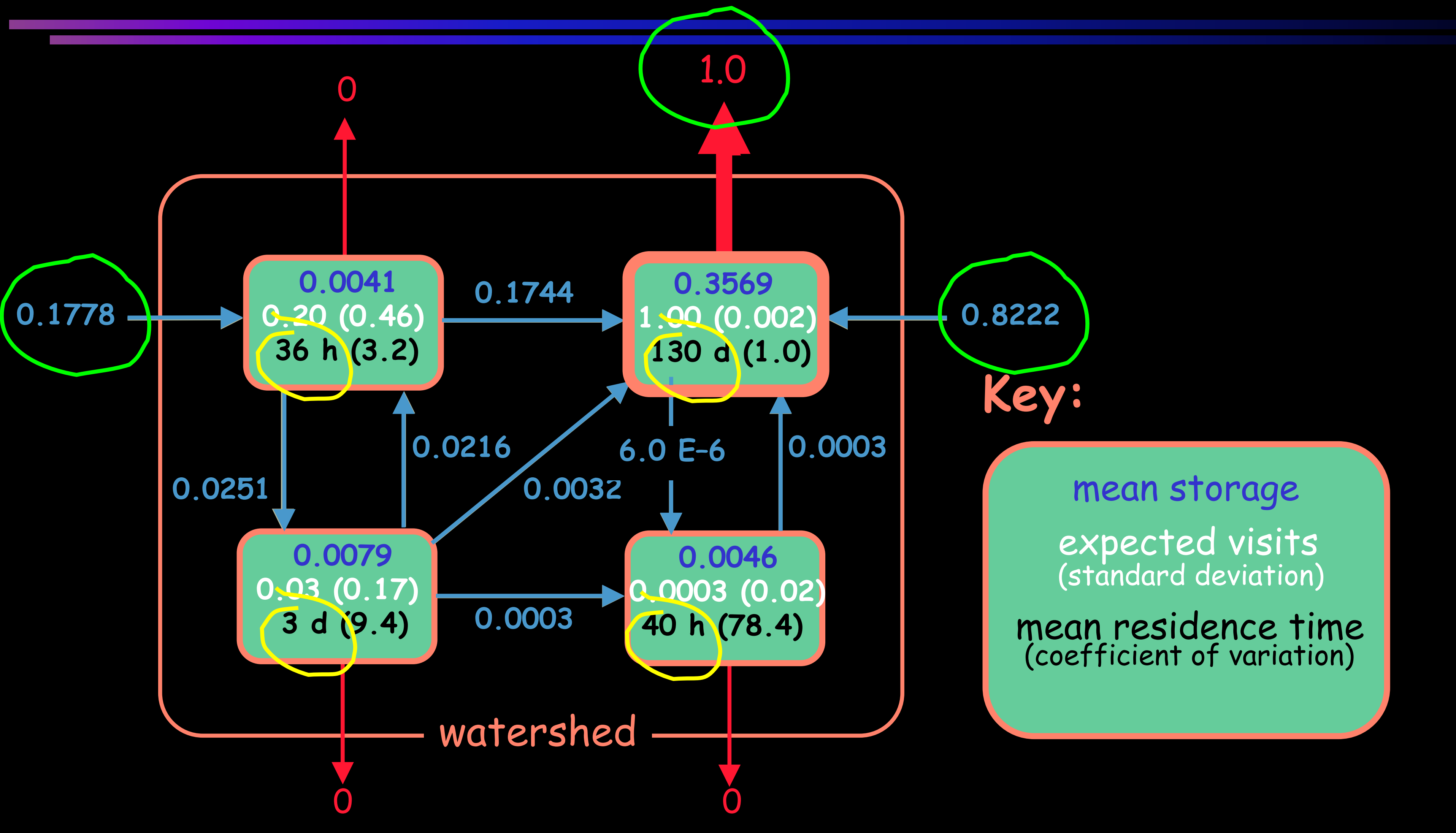
Okekenokee Water Model

Unit Input Environ, Upland Ground Water



Okekenokee Water Model

Unit Input Environ: Swamp Surface Water



Structural Environ Analysis
Environ Intersection

Premise:

A necessary condition for electromagnetic interaction between a binary pair of objects within systems is intersection of their output and input environs: $E_j 1 E_i'$.

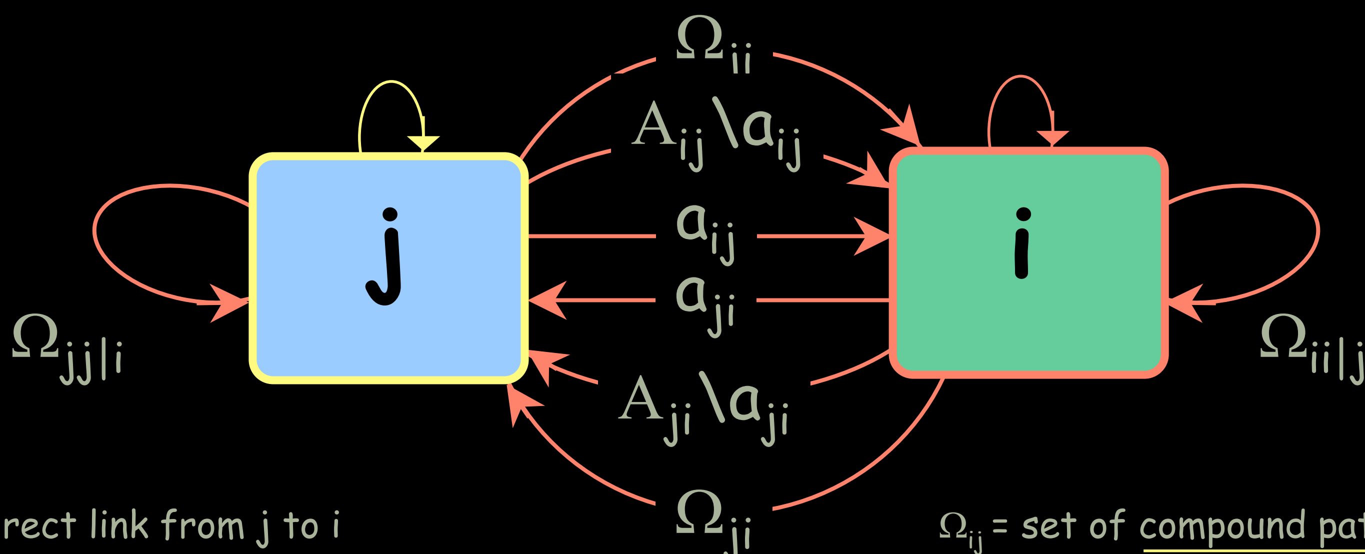
That is, for node j in a network to transfer energy or matter to node i …

j must be in the input environ E_i' of i, and
i must be in the output environ E_j of j.

With such a configuration, a blueprint of pathways encompassing five transport modes can be identified at and within the boundary of the overlapping environs …

Structural Environ Analysis

Blueprint for j to i transfers in $E_j 1 E_i'$



a_{ij} = direct link from j to i

a_{ji} = direct link from i to j

$A_{ij}\backslash a_{ij}$ = set of simple paths[1] from j to i except a_{ij}

$A_{ji}\backslash a_{ji}$ = set of simple paths from i to j except a_{ji}

Ω_{ij} = set of compound paths[2] from j to i

Ω_{ji} = set of compound paths from i to j

$\Omega_{jj|i}$ = cycles at originating node j

$\Omega_{ii|j}$ = cycles at terminating node j

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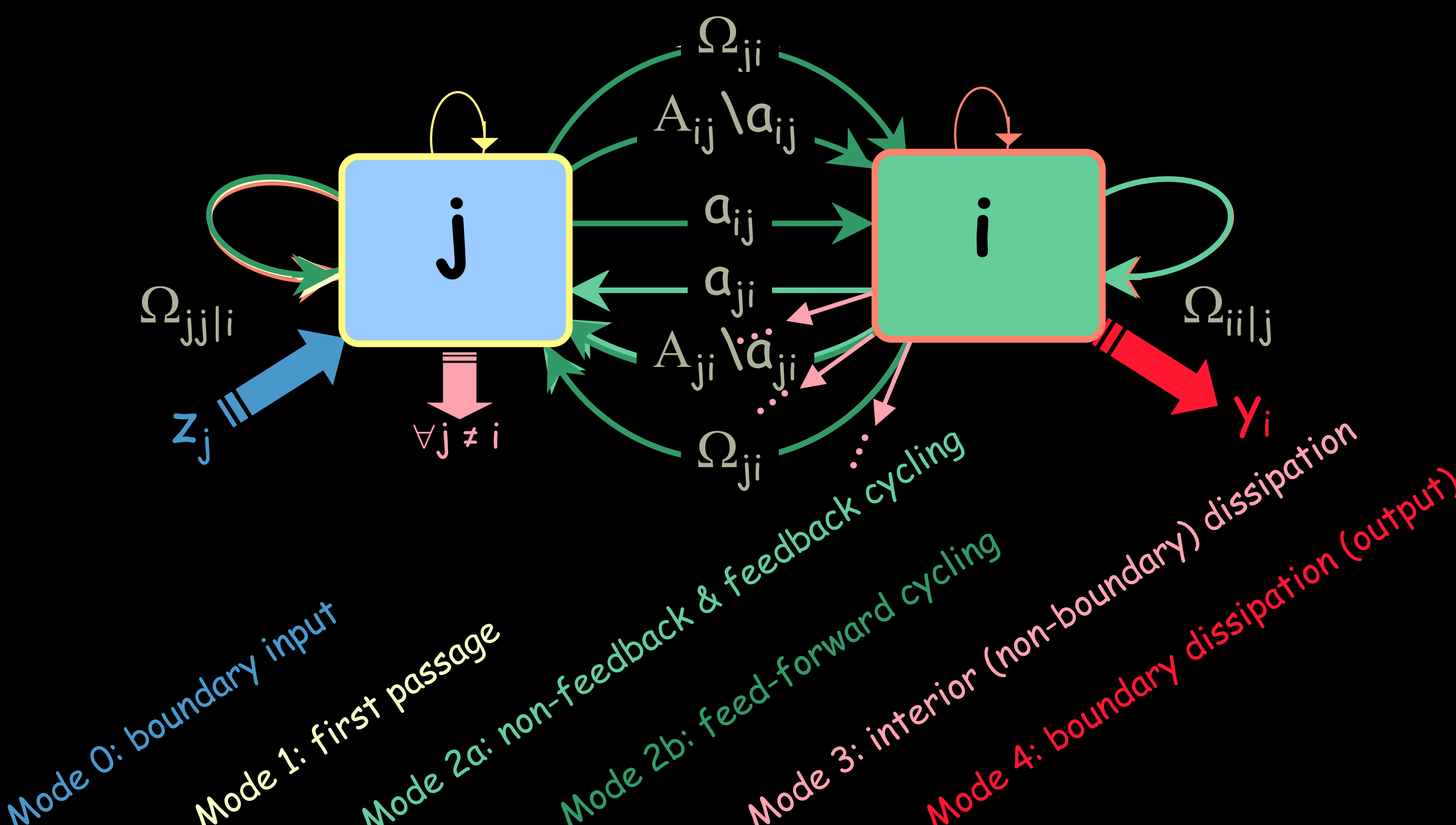
[1] without repeating nodes

[2] with repeating nodes

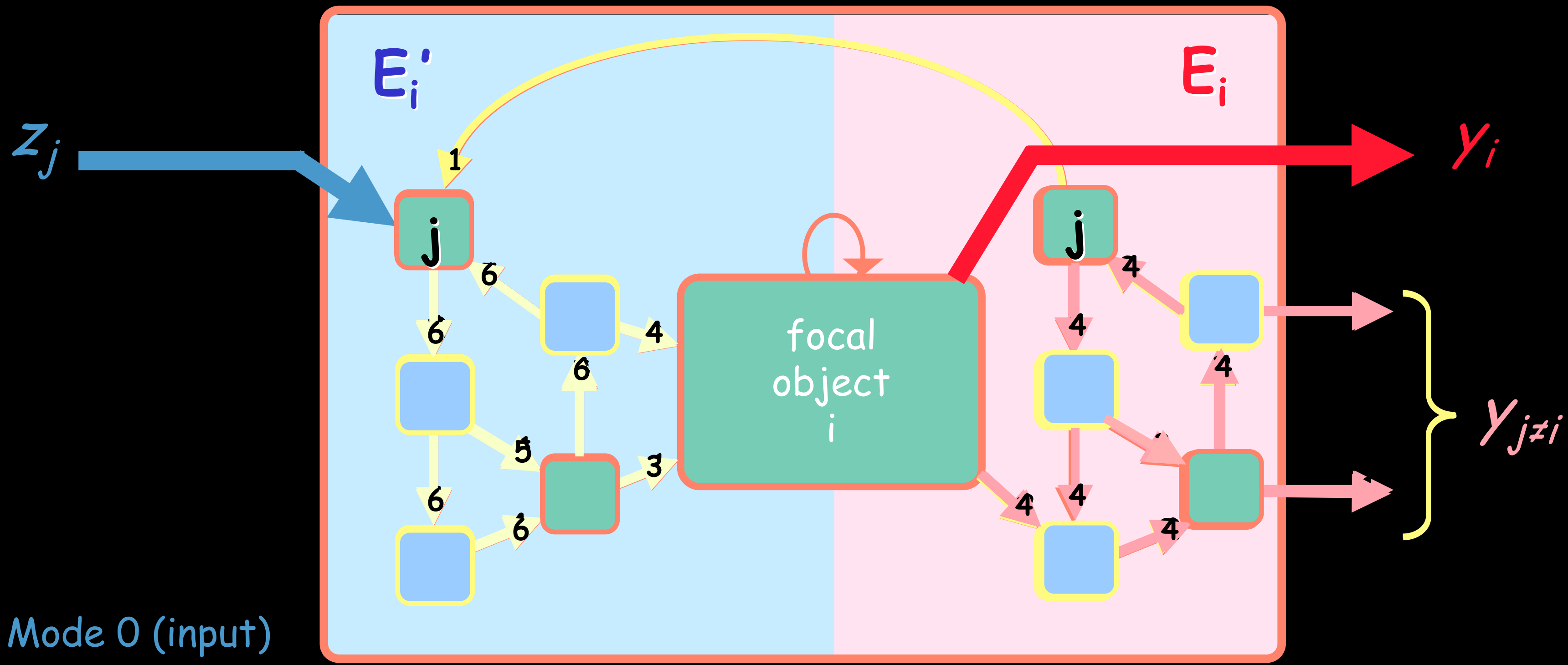
Structural Environ Analysis

Five Modes of j to i transfer in $E_j\mathbf{1}E_i'$



Structural Environ Analysis

Five Pathway Modes in Environs



Mode 0 (input)

Mode 1 (1st passage): 1.1, 1.2, 1.3, 1.4
1.5 v (1st passage), 1.6 v (1st passage)

Mode 2 (cycling): 2.1 v (1st passage)

Mode 3 (indirect dissipation): 3.1, 3.2
3.3 v (3.1 w 3.2), 3.4 v (3.1 w 3.2)

Mode 4 (direct output)