NON-LINEAR METHODS FOR RANKING QUALITATIVE NON-MONOTONE DECISION PREFERENCES

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ABSTRACT

One approach to option ranking in qualitative decision making is first to automatically construct a quantitative evaluation model using the qualitative and quantitative (QQ) method and then use the quantitative model for ranking. However, the quantitative model constructed by QQ, which uses linear techniques, fails to provide consistent and complete option ranking of non-monotone decision preferences. In this paper we investigate alternative methods for consistent and complete option ranking of non-monotone preferences by using non-linear techniques. Results show that non-linear methods are superior to linear techniques, when dealing with non-monotone two-attribute decision problems.

1 INTRODUCTION

Qualitative decision problems appear in our everyday life all the time. We manage to successfully decide in cases when we have to make a few qualitative decisions. However, when we have to make many decisions, we saturate in a way that we cannot make consistent decisions for all possible situations that occur. It was shown that when dealing with problem of classification, humans face the natural upper limit capacity to distinguish among five different classes [1]. Trained users may distinguish among seven different classes, and really highly skilled users may achieve to distinguish nine different levels. This limit to distinguish up to nine classes, constrains us in the way we perform decision making when faced with the problem of evaluating many options. Additionally, when we evaluate many qualitative options, it often happens that several options belong to the same qualitative output class which means that they are almost equally preferred [2]. In order to distinguish among the options that belong to the same class, ranking of options within classes has to be performed. Therefore, in order to consistently rank qualitative options we seek for algorithms that would support our decisions in the process of qualitative decision making.

2 DESCRIPTION OF THE PROBLEM

One of the methodologies that deals with qualitative multi attribute decision problems is DEX [3]. Attributes in DEX are represented with discrete or qualitative values, while the inference is presented with if-then decision rules given in tabular format. In addition to the qualitative description of options in DEX, we need a numeric utility for ranking of options that belong to the same qualitative class. For this purpose, we use combined qualitative and quantitative (QQ) method [4] to obtain the numerical utility. To find the numerical utility in QQ, first a mapping of qualitative variable into quantitative variable is performed. In this process, each of the values of a qualitative variable is substituted with ordinal numbers. For example, let a qualitative variable has preferentially ordered values such as {good, better, the best}, where the decision maker has the preference of "the best" $\succ$ "better" $\succ$ "good", and where the sign $\succ$ denotes "is strictly more preferable than". Then the qualitative values are substituted with ordinal numbers, for instance {1,2,3}, where number 1 represents "good", number 2 represents "better" and number 3 represents "the best". The variables in the quantitative domain are compared with the relation “is greater than" or simply >. This mapping ensures that the greater the numerical value, the larger the preference of the decision maker.

The next step is to quantitatively evaluate the options in a way that the evaluated ranking describes the preferences of the decision maker as precisely as possible. To quantitatively evaluate the options within classes, we use the additive value function, which is a well-known method and it is easily understandable. It has the form of

$$y_i = \sum_i \omega_i \alpha_i \qquad (1)$$

where the coefficients ω_i are weighting factors (weights) and α_i are values of attributes. Such linear functions are used in many areas like in economy, commerce and operational research. For instance, the entire theory of linear programming is based on the assumption that decision makers' preferences may be represented by a linear value function [5]. The main problem with this kind of representation is how to choose the weights properly so that we can correctly rank the options that describe the decision maker's preferences. Different methods carry out this task differently, as described below in section 4.

3 EXPERIMENTAL SETUP

In our study, we have evaluated several methods for determining the weights from numerical tables that represent the decision rules used in DEX methodology. These rules are represented in the form of decision tables (DTs) that have the format given in Table 1. The first column in the decision table is the number of the option that has to be evaluated; the second and third columns are values of the two attributes of the option; the fourth column is the class to which the option belongs. All the remaining columns are evaluations of the options obtained with different methods. In our experiment, all the numeric attributes in the decision tables may acquire three different values: {1,2,3}.

Each decision table comprises all the possible combinations of the attribute values, i.e., without missing options. Each decision table represents a possible decision maker's preference, from which we try to determine the weights of the evaluation model (1).

4 OVERVIEW OF THE USED METHODS

To determine the weights, in our study we examined the performance of the following methods: QQ (qualitative-quantitative approach), different definitions of Gini index (Gini index defined by Breiman which we refer further as Gini; Gini Covariance; Gini based on the population which we refer as Gini Population), Information Gain and χ^2.

4.1 QQ Method

The Qualitative-Quantitative or QQ method [4] maps qualitative attributes into quantitative and then uses multiple linear regression to determine the weights of the additive value function. It first calculates the value of weights ω_i by using the relation

$$g = \sum_i \omega_i \alpha_i + \omega_0 \qquad (2)$$

and then it constraints the outputs of the options into intervals $c \pm 0.5$, where c is the class to which the output belongs. The final output of the QQ method is a set of functions. For each class c, the corresponding ranking function is

$$f_c = n_c g + k_c \qquad (3)$$

Here, n_c and k_c are parameters that are different for each class and that ensure that the final output of the function is in the interval $c \pm 0.5$ This means that qualitative and quantitative evaluations are always consistent: if an option belongs to a qualitative class c, then the numerical evaluation is in the interval $c \pm 0.5$. That way, when we look at a certain numerical evaluation, we immediately know the class of the option (except for borderline numerical evaluations, such as 2.5).

There are two important properties of quantitative rankings: completeness and consistency. The ranking is *complete* if there are no two options that receive the same evaluations, so that the options can be uniquely ranked from best to worst. To determine the *consistency* of ranking, we observe the differences between two options. For all pairs of options whose values of attributes differ by the same amount (i.e., the same vector), the signs of the difference of their numerical evaluations have to be the same.

The QQ method has been designed to cope with decision tables that are monotone (the class always increases or remains constant with the increasing values of attributes) and close to linear (they can be sufficiently well approximated by a linear function). Therefore, the disadvantage of QQ is that in general it cannot consistently rank non-monotone decision tables. For this reason, we have to look for other methods to perform consistent ranking.

4.2 Gini Coefficient, Information Gain and χ^2

The Gini coefficient (or Gini index) was first proposed by Italian statistician Corrado Gini in 1912 as a measure of income inequality [6]. It is mathematically defined as a ratio between the Lorenz curve that plots the income of population versus population and perfect equality of income. In later works it is defined as second order of Shannon's Entropy [7]. Since its first proposal, the Gini index has been used in many different areas to measure different kinds of distributions. In machine learning it is used for making splits in decision trees [8] and for representation of the performers of different classifiers [9].

In this paper we examine different estimates for Gini index: the definition of Gini index as introduced by Breiman et al. [8], the Gini covariance approach [6] and Gini population approach [10].

Information gain has its origin in information theory [11] and it is frequently used in decision tree learning for determining the attribute that gives most information regarding some splitting criteria. It is defined as the difference between the original information and the information obtained after using an attribute to split the decision tree.

χ^2 distribution has its origin in statistics and was devised as a test of goodness of fit [12] of an observed distribution to a theoretical one.

In this paper we exploited the Gini index, Information Gain and χ^2 for the calculation of weights ω_i in (1) for non-monotone decision tables. Unlike QQ, which uses multiple linear regression for determining weights ω_i in (2), these methods use non-linear calculations to obtain some measure of influence of each of the attributes on the output class. This measure is used to determine the weights ω_i in (1). As soon as the weights are obtained, the evaluation and ranking of options is exactly the same as in QQ. We continue to determine the value of the function y_i as given in (1), and finally we constrain the output rankings to the interval $c \pm 0.5$ by using (3).

5 RESULTS AND DISCUSSION

We evaluated the selected methods on a complete set of all the decision tables which map two three-valued attributes (x_1 and x_2) to a three-valued class – in total, there are 3^9=19.683 different tables.

As evaluation criteria, we used the performance of consistent rankings of each method within classes. However, there are decision tables for which exist multiple consistent and complete ranking solutions. Such a decision table with two ranking solutions is presented in Table 5. Namely, in Table 5 in class two, ranking of options with numbers 4, 5 and 6 is different when Gini Population method is used compared to the one when other methods are used. In that case, as evaluation criteria we use the sum of relative absolute error (RAE) over all table rows. We choose as the best method, the one with the smallest RAE.

Although QQ method was originally developed for ranking monotone decision tables, we evaluated its performances on all the decision tables. Results show that QQ provides a complete ranking of 13 % of the whole set of decision tables. The rest of the decision tables are not consistently and/or completely ranked with QQ.

Ranking with Gini methods provides better results than ranking with QQ for non-monotone cases, however there are differences in rankings depending on the used estimator for the Gini methods. In general, ranking with Information Gain and χ^2 is better than ranking with QQ, Gini and Gini Covariance, but worse that ranking with Gini Population method. The percentage of completely ranked decision tables with each method is given in Figure 1.

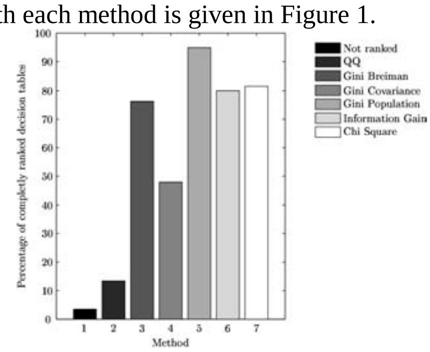

Figure 1: *Distribution of DTs that are completely ranked with different methods*

As shown in Figure 2, results are divided into five groups:

- Group 1: decision tables that cannot be completely ranked by any of the methods (an example is given in Table 1 and calculated weights are given in Table 2);
- Group 2: decision tables that are completely and consistently ranked only by Gini Population method (example of a decision table of this kind is given in Table 3 and calculated weights are given in Table 4);
- Group 3: decision tables that are completely and consistently ranked with QQ method, but also with other methods;
- Group 4: decision tables that are improperly ranked by QQ, but all other methods perform flawlessly (an example is given in Table 5 and weights in Table 6);
- Group 5: decision tables that are completely and consistently ranked only by Gini, Information Gain and χ^2.

For the decision tables in group 1 we have to look for algorithms other then those included in this research. These decision tables have in common that:

Table 1: *Example of DT for which all methods fail to provide complete ranking*

No.	x_1	x_2	class	All methods
1	1	1	1	0.83
2	2	1	1	1.16
3	1	2	1	1.16
4	3	1	2	2.00
5	2	2	2	2.00
6	1	3	2	2.00
7	3	2	3	2.83
8	2	3	3	2.83
9	3	3	3	3.16

Table 2**:** *Weights obtained for the DT given in* Table 1

weights	QQ	Other methods
ω_0	33.33	n/a
ω_1	33.33	50
ω_2	33.33	50

Table 3**:** *Example of DT for which complete ranking is possible only with the method Gini Population*

No.	x_1	x_2	class	Gini Population	All other methods
1	1	1	1	0.73	1.00
2	1	2	1	1.00	1.00
3	1	3	1	1.26	1.00
4	2	1	2	1.69	1.75
5	2	2	2	1.91	1.75
6	3	3	2	2.30	2.25
7	3	1	3	2.79	3.25
8	3	2	3	3.12	3.25
9	2	3	3	3.20	2.75

Table 4: *Weights obtained for the DT given in* Table 3

weights	QQ	Gini Population	Other methods
ω_0	0.00	n/a	n/a
ω_1	71.42	56.74	0
ω_2	28.57	43.25	100

Table 5: *Example of DT for which only Gini Population provides different complete ranking compared with other methods and RAE is smallest for Gini Population*

No.	x_1	x_2	class	Gini Population	Information Gain	Other methods
1	1	1	1	0.73	0.80	0.81
2	1	2	1	1.00	1.00	1.00
3	1	3	1	1.26	1.19	1.18
4	2	1	2	1.81	1.79	1.79
5	3	1	2	2.09	2.20	2.20
6	2	2	2	2.18	1.98	1.95
7	3	2	3	2.81	3.01	3.04
8	2	3	3	2.90	2.79	2.79
9	3	3	3	3.18	3.20	3.20

Table 6: *Weights obtained for the DT given in* Table 5

weights	QQ	Gini Population	Information Gain	Other methods
ω_0	22.22	n/a	n/a	n/a
ω_1	55.55	56.74	31.36	28.57
ω_2	22.22	43.25	68.63	71.42

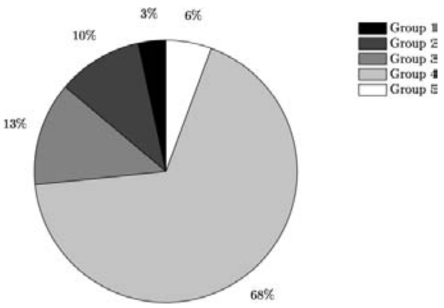

Figure 2: *Distribution of DTs ranking results in five groups*

- the methods provide equal weights for the two attributes or
- the methods choose that only one attribute is important and weight that attribute 100 %, while they weigh the second attribute 0 %.

As a general instruction for ranking of decision tables with two three-valued attributes, we propose:

- to first rank using Gini Population method, and
- if Gini Population fails, rank with any of the three methods: Gini, Information Gain or χ^2 .

If none of the above provides a complete ranking, then the decision table belongs to the small group of decision tables that are not fully ranked with any of the discussed methods. In this case, we have to accept the incomplete ranking or seek for a different approach.

6 CONCLUSION

In this work we modeled 19.683 decision tables that consist of two three-valued attributes and three-valued class, using 6 different non-linear techniques for determining the weights of the additive weighting function model. We have shown that the QQ method may be used for ranking in 13 % of the whole set of decision tables. We manage to rank the options in most of the decision tables when weights in the model are determined by using different estimates of Gini coefficients, from which the most prominent one is Gini Population. Additionally, for one group of decision tables, the full ranking is possible only when using Gini, Information Gain and χ^2 . Furthermore, when multiple complete rankings exist for a DT, we propose to use the one with smalles RAE. In addition 3 % of the set of non-monotone decision tables are not fully ranked with any of the methods. For them we have to further investigate other methods. In future we want to investigate the applicability of the described methods for different kinds of decision tables, for example for decision tables with more than two attributes, with different domains of the attributers and different number of options.

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References

[1] T.L.Saaty, Why the magic number seven plus or minus two, *Math. Comput. Modell.*, vol 38. pp 233-244, 2003.
[2] M. Bohanec, *Odločanje in modeli. DFMA*, 2006.
[3] M. Bohanec, DEXi: Program for multi-attribute decision making, User's manual, Version 3.00. IJS Report DP-9989, Jožef Štefan Institute, Ljubljana, 2008, http://kt.ijs.si/MarkoBohanec/pub/DEXiManual30r.pdf
[4] M. Bohanec, B. Urh, V. Rajkovič, Evaluation of options by combined qualitative and quantitative methods, *Acta Psychol*, vol. 80, pp. 67-89, 1992.
[5] S. French, *Decision theory: an introduction to the mathematics of rationality*. NY, Halsted Press, 1986.
[6] K. Xu, How has the literature on Gini index evolved in the past 80 years, Dalhousie University, Department of Economics, 2004.
[7] L. Wehenkel, On uncertainty measures used for decision tree induction, *P IPMU*, pp. 413-418, Granada, Spain 1996.
[8] L. Breiman, J. H. Friedman, R. A. Olshen, and C. J. Stone, *Classification and Regression Trees*, Belmont, California, USA Wadsworth Publ Company, 1984.
[9] D. J. Hand, R. J. Till, A simple generalization of the area under the ROC curve for multiple class classification problems, *Mach Learn*, vol. 45 no. 2, pp. 171-186, 2001.
[10] F. Noorbakhsh, International convergence or higher inequality in human development? Evidence for 1975-2002, in (G. Mavrotas and T. Shorrocks, eds.), *Adv Dev: Core Thms in Glob Econ*, pp. 149-167, 2007
[11] L. E. Raileanu, K. Stofiel, Theoretical comparison between the Gini index and Information Gain criteria, *Ann Math Artif Intel*, vol. 41, pp. 77-93, 2000.
[12] R. A. Fisher, On a Distribution Yielding the Error Functions of Several Well Known Statistics, *P Int Congr Math, Toronto*, vol. 2, pp 805-813, 1924.

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