EMPIRICAL COMPARISON OF THREE METHODS FOR APPROXIMATING DEX UTILITY FUNCTIONS

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Abstract: DEX is a qualitative multi-criteria decision analysis method. It provides support to decision makers in evaluating and choosing decision alternatives, using discrete attributes and rule-based utility functions. This work builds upon our previous attempt of approximating DEX utility functions with methods UTA and ACUTA, aimed at improving the sensitivity of qualitative models and providing an interpretation of DEX utility functions. In this work we empirically compare three methods for approximating qualitative DEX utility functions with piecewise-linear marginal utility functions: Direct marginals, UTADIS and Conjoint analysis. The results show that these methods can accurately approximate complete, monotone DEX utility functions.

Keywords: decision support, multi-criteria decision making, utility function, DEX, UTADIS, conjoint analysis, direct marginals method

1 INTRODUCTION

Multi criteria decision analysis (MCDA) [7] deals with solving decision problems involving multiple, possibly conflicting, criteria. It provides a number of methods to create decision models by using information provided by the decision maker. Provided information can be given in various forms, using different representations. Converting representations from one form to another is often highly desirable, as it can bridge the gap between different methodological approaches and enrich the capabilities of individual ones.

At a general level, this study addresses two types of utility function representations, qualitative and quantitative, and investigates how to convert the former to the latter. At a specific level, we compare three methods of approximating DEX utility functions by piece-wise linear marginal utility functions: the Direct marginals method, UTADIS and the Conjoint analysis method. DEX [5] is a qualitative MCDA method, which employs discrete attributes and discrete utility functions defined in a rule-based point-by-point way (see section 2.1). This makes DEX suitable for classifying decision alternatives into discrete classes. The Direct marginals method (section 2.3) establishes marginal utility functions by a projection of a DEX utility function to individual attributes. UTADIS [6] (section 2.4) is a quantitative method that constructs numerical additive utility functions from a provided subset of alternatives and assigns this alternatives to predefined ordered groups. Conjoint analysis [8] (section 2.5) is a method that constructs numerical additive utility functions through determining attribute importance, the appropriate importance levels and the effects of combining different attributes on the measured variable. The three methods were experimentally assessed on a collection of artificially generated complete monotone DEX utility functions.

All three methods are aimed at providing an approximate quantitative representation of a qualitative DEX function. This extends the capabilities of DEX and is useful for several reasons. First, the newly obtained numerical evaluations facilitate an easy ranking and comparison of decision alternatives, especially those that are assigned the same class by DEX. Consequently, the sensitivity of evaluation is increased. Second, the sheer form of numerical functions may provide additional information about the properties of underlying DEX functions, which is useful in verification, representation and justification of DEX models. In this study, we focus on the accuracy of representation.

There have been several previous attempts to approximate DEX utility functions. A linear approximation method is commonly used in DEX to assess criteria importance [3]. An early method for ranking of alternatives and improving the sensitivity of evaluation called QQ [12] has been proposed in [2]. Recently, extensive research has been carried out to approximate DEX functions with copulas [12]. This paper builds upon our previous work on approximating DEX utility functions by using methods UTA and ACUTA [11]. The methods used in the present study were chosen because they do not have convergence issues when approximating discrete functions as opposed to the methods tried in [11].

2 METHODS

2.1 DEX method

DEX [5] is a qualitative MCDA method for the evaluation and analysis of decision alternatives, and is implemented in the software DEXi [4]. In DEX, all attributes are qualitative and can take values represented by words, such as *low* or *excellent.* Attributes are generally organised in a hierarchy. The evaluation of decision alternatives is carried out by utility functions, which are represented in the form of decision rules.

In the context of this paper, we focus on individual utility functions. For simplicity, we assume that all attributes are ordinal and preferentially ordered, so that a higher ordinal value represents a better preference. In this setting, a DEX utility function f is defined over a set of attributes $\vec{x} = (x_1, x_2, \ldots, x_n)$ so that

$$f : X_1 \times X_2 \times \cdots \times X_n \to Y$$

Here, $X_i, i = 1, 2, \ldots, n$, denote value scales of the corresponding attributes x_i, and Y is the value scale of the output attribute y:

$$X_i = \{1, 2, \ldots, k_i\}, i = 1, 2, \ldots, n \quad \text{and} \quad Y = \{1, 2, \ldots, c\}$$

The function f is represented by a set of decision rules

$$F = \{(\vec{x}, y) | \vec{x} \in X_1 \times X_2 \times \cdots \times X_n, y \in Y, y = f(\vec{x})\}$$

Each rule $(\vec{x}, y) \in F$ defines the value of f for some combination of argument values $\vec{x}$. In this study, we assume that all functions are *complete* (defined for all combinations of argument values) and *monotone* (when argument values increase, the function value increases or remains constant).

2.2 Approximation of DEX utility functions

All methods assessed in this study are aimed at approximation of some DEX utility function f with marginal utility functions $u_i : X_i \to \mathbb{R}, i = 1, 2, \ldots, n$. The functions u_i are assumed to take a piece-wise linear form: the numeric value of $u_i(v)$ is established from f for each $v \in X_i$, while its value for $v \notin X_i$ is linearly interpolated from the closest neighbouring points.

On this basis, f is approximated as a weighted sum of marginal utility functions:

$$u(x) = u(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} \omega_i u_i(x_i)$$

Here, $\omega_i \in \mathbb{R}, i = 1, 2, \ldots, n$, are weights of the corresponding attributes, normalised so that $\sum_{i=1}^{n} \omega_i = 1$.

2.3 Direct marginals method

The direct marginals method establishes the marginal utility function $u_i(v)$ as an average value of target attribute y for decision rule $a \in F$, where $x_i(a) = v$. Let $F_{i,v} \subset F$ denote all decision rules where $x_i(a) = v$. Then

$$u_i(v) = \frac{1}{|F_{i,v}|} \sum_{\{a \in F \ | \ x_i(a)=v\}} y(a), \quad i = 1, 2, \ldots, n, \quad v \in X$$

In the experiments (section 2.6), all functions $u(x)$ were scaled to the $[0, 1]$ interval, therefore importance weights for attributes were computed as a percentage of total utility range covered by the range of a particular attribute.

2.4 UTADIS method

The UTADIS method [6] is an extension of UTA (UTilités Additives) method [9] that enables decision maker to assign alternatives to predefined ordered groups. Thus it is very well suited to our problem of approximating discrete DEX functions, assuming that each DEX decision rule $a \in F$ defines some (hypothetical) decision alternative. UTADIS approximates u_i as:

$$u_i(x_i(a)) = u_i(x_i^J) + \frac{x_i(a) - x_i^J}{x_i^{J+1} - x_i^J}[u_i(x_i^{J+1}) - u_i(x_i^J)]$$

It is assumed that each alternative values are divided to $(\alpha_i - 1)$ equally sized intervals $[g_i^J, g_i^{J+1}]$. The alternatives are assigned to groups by using thresholds t_i: $u(x_j) \geq t_1 \Rightarrow a \in C_1,\ t_2 \leq U(g_j) < t_1 \Rightarrow a \in C_2, \ldots, U(g_j) < t_{c-1} \Rightarrow a \in C_c$.

UTADIS searches for marginal utility functions by solving the linear programming problem $min\ E = \sum_{k=1}^{c} \frac{\sum_{a_j \in C_k} \sigma(a)_j^+ + \sigma(a)_j^-}{m_k}$, where σ^+, σ^- denote errors after violation of upper/lower bound of a group C_k and m_k denotes a number of alternatives assigned to the group C_k.

2.5 Conjoint analysis method

Conjoint analysis [8] is designed to explain decision maker's preferences. It outputs attribute importance, their interactions and utility functions for each attribute in a decision making problem. The original decision table is transformed in a binary matrix x_b, that encodes the original attribute values by using a fixed number of bits. This matrix is used to compute a matrix of deviation scores $x = x_b - \mathbb{1}\mathbb{1}^\tau x_b(\frac{1}{n})$. The utility value is computed as $b = (x^\tau x)^{-1} \cdot (x^\tau y)$, where y denotes a vector containing deviation scores of the target variable. Attribute importance is obtained by observing the percentage of total utility range covered by the range of a particular attribute.

2.6 Experimental procedure

The goal of experiments was to assess and compare the performance of the three methods – Direct marginals, UTA, and Conjoint analysis – on artificially generated, complete, and monotone DEX utility functions. For this purpose, we generated all monotone functions for spaces with dimensions $3 \times 3 \rightarrow 4$, $3 \times 4 \rightarrow 3$, $4 \times 4 \rightarrow 3$ and $5 \times 6 \rightarrow 7$ (The notation $3 \times 3 \rightarrow 4$ denotes the space of all utility functions having two three-valued arguments, that map to 4 values). Evaluation was also performed on several randomly generated function sets of different sizes: $3 \times 4 \times 3 \times 5 \rightarrow 6$, $4 \times 5 \times 5 \rightarrow 6$, $5 \times 6 \rightarrow 7$, $6 \times 7 \rightarrow 7$, $8 \times 7 \rightarrow 7$ containing 1000 functions, and $3 \times 5 \times 3 \times 4 \rightarrow 4$ containing 100 functions.

The experimental procedure consisted of predicting the target utility function for all the generated functions by using three selected methods, and computing evaluation scores for each method's resulting utility function. Two measures were used for evaluation: the Area Under the Curve (AUC) and the Root Mean Squared Error (RMSE). Finally, we computed the average of AUC and RMSE with corresponding standard deviation for sets of functions with given dimensions to compare method performance on the whole function set. Since these methods compute utility values in different ranges, all the functions were scaled to the $[0, 1]$ interval.

All experiments were performed in R programming language by using 'MCDA' [10], 'conjoint' [1] and 'pROC' [13] R packages. In addition, we implemented Direct marginals method, the RMSE measure, monotone function generator that generates all monotone functions in some space with given dimensions, and a random monotone function generator that generates a number of random monotone functions in a space with given dimensions.

3 RESULTS

In this section we present results of approximating DEX utility functions with methods Direct marginals, Conjoint analysis and UTADIS. A thorough evaluation can be seen in Table 1.

method	space dimension	num.	avg. AUC	avg. RMSE	succ.
Direct marginals	$3 \times 3 \to 4$	979	0.996 ± 0.015	0.532 ± 0.246	100%
	$3 \times 4 \to 3$	489	0.998 ± 0.011	0.404 ± 0.162	100%
	$4 \times 4 \to 3$	2014	0.995 ± 0.013	0.416 ± 0.135	100%
	$5 \times 6 \to 7$	1000	0.981 ± 0.021	0.981 ± 0.354	100%
	$6 \times 7 \to 7$	1000	0.978 ± 0.021	1.0 ± 0.327	100%
	$8 \times 7 \to 7$	1000	0.975 ± 0.023	0.980 ± 0.308	100%
	$4 \times 5 \times 5 \to 6$	1000	0.945 ± 0.025	1.056 ± 0.245	100%
	$3 \times 4 \times 3 \times 5 \to 6$	1000	0.921 ± 0.027	1.145 ± 0.225	100%
	$3 \times 4 \times 5 \times 3 \times 4 \to 4$	100	0.928 ± 0.018	0.818 ± 0.101	100%
Conjoint analysis	$3 \times 3 \to 4$	979	0.989 ± 0.026	0.564 ± 0.236	100%
	$3 \times 4 \to 3$	489	0.990 ± 0.026	0.416 ± 0.159	100%
	$4 \times 4 \to 3$	1763	0.987 ± 0.025	0.423 ± 0.132	100%
	$5 \times 6 \to 7$	1000	0.971 ± 0.027	1.014 ± 0.329	100%
	$6 \times 7 \to 7$	1000	0.967 ± 0.028	1.023 ± 0.305	100%
	$8 \times 7 \to 7$	1000	0.964 ± 0.029	0.996 ± 0.291	100%
	$4 \times 5 \times 5 \to 6$	1000	0.925 ± 0.033	1.056 ± 0.233	100%
	$3 \times 4 \times 3 \times 5 \to 6$	1000	0.896 ± 0.035	1.142 ± 0.214	100%
	$3 \times 4 \times 5 \times 3 \times 4 \to 4$	100	0.904 ± 0.028	0.808 ± 0.102	100%
UTADIS	$3 \times 3 \to 4$	979	0.970 ± 0.063	0.722 ± 0.293	99.7%
	$3 \times 4 \to 3$	489	0.976 ± 0.064	0.567 ± 0.215	99.6%
	$4 \times 4 \to 3$	1763	0.972 ± 0.069	0.567 ± 0.212	99.9%
	$5 \times 6 \to 7$	1000	0.931 ± 0.065	1.569 ± 0.688	100%
	$6 \times 7 \to 7$	1000	0.924 ± 0.064	1.574 ± 0.646	100%
	$8 \times 7 \to 7$	1000	0.916 ± 0.068	1.545 ± 0.672	100%
	$4 \times 5 \times 5 \to 6$	1000	0.898 ± 0.054	1.299 ± 0.389	100%
	$3 \times 4 \times 3 \times 5 \to 6$	1000	0.880 ± 0.049	1.292 ± 0.312	100%
	$3 \times 4 \times 5 \times 3 \times 4 \to 4$	100	0.911 ± 0.034	0.875 ± 0.151	100%

Table 1: Comparison results for the Direct marginals, Conjoint analysis and UTADIS method on various generated DEX monotone utility functions. For each method and space dimensions, the columns show the number of utility functions (num.), average AUC and RMSE with standard deviation, and the percentage of successfully approximated functions (succ.).

The results from Table 1 show that all three methods can approximate the majority of artificially created complete monotone DEX utility functions; only UTADIS returns errors when faced with trivial functions (containing equal target value for every alternative), which is likely a problem of implementation. The Direct marginals method achieved the best evaluation score on all tested functions in both AUC and RMSE measures and is closely followed by the Conjoint analysis method. UTADIS method has somewhat lower results and higher standard deviation. The results indicate that the AUC value decreases with the increase of function domain dimensions and the cardinality of attribute value set for all tested methods (see Figure 1). Results for the RMSE measure are little less conclusive. The error rises slowly for the Conjoint analysis and Direct marginals method but drops for UTADIS method. AUC increase and RMSE decrease on the last dataset could be caused by a small generated function sample (100 random functions) and the fact that the target attribute could have only 4 different values.

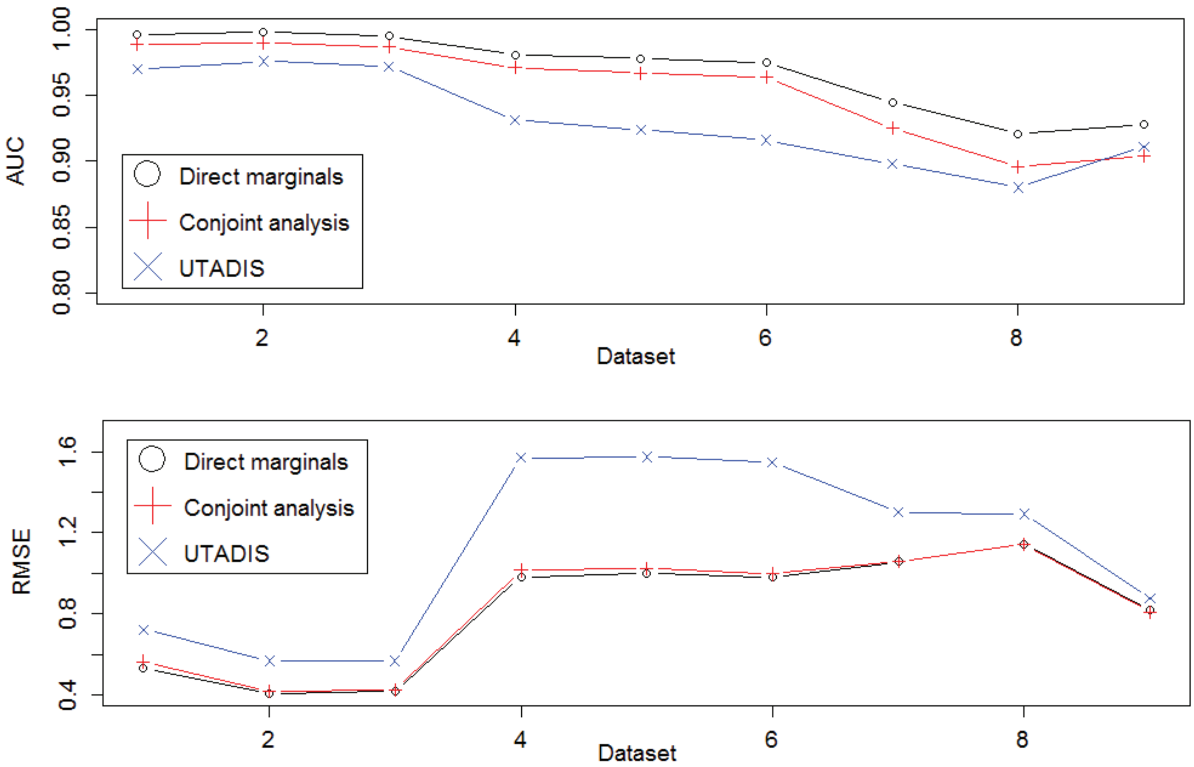

Figure 1: AUC and RMSE comparison for all three methods. Function sets are presented in the same order as in Table 1.

4 CONCLUSION

In this work we presented a new method for approximating monotone DEX utility functions, the Direct marginals method, and compare its performance with two known decision support methods: UTADIS and Conjoint analysis. The methods were evaluated on several sets of randomly generated functions with domains of different dimensions and the resulting utility functions were scaled to the $[0, 1]$ interval, to allow comparative analysis. The overall quality of approximation is assessed by using multi-class AUC and RMSE measures. The Direct marginals method outperformed other approaches on all test functions with respect to the AUC method and on majority of test functions with respect to the RMSE measure. Conjoint analysis follows very closely. All tested methods give fairly good approximations of monotone DEX utility functions and give additional insight into decision makers preferences on attribute level, but also between different attributes. We believe that such insight might be useful for different decision problems, for instance, product manufacturers to evaluate their products and locate important and interesting features that should be improved or changed to satisfy their customers.

In the future work, we would like to address the problem of approximating incompletely defined DEX functions and DEX functions defined with distribution of classes.

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